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APPICATIONS OF GAUSS'S LAW
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APPLICATION'S OF GAUSS'S LAW

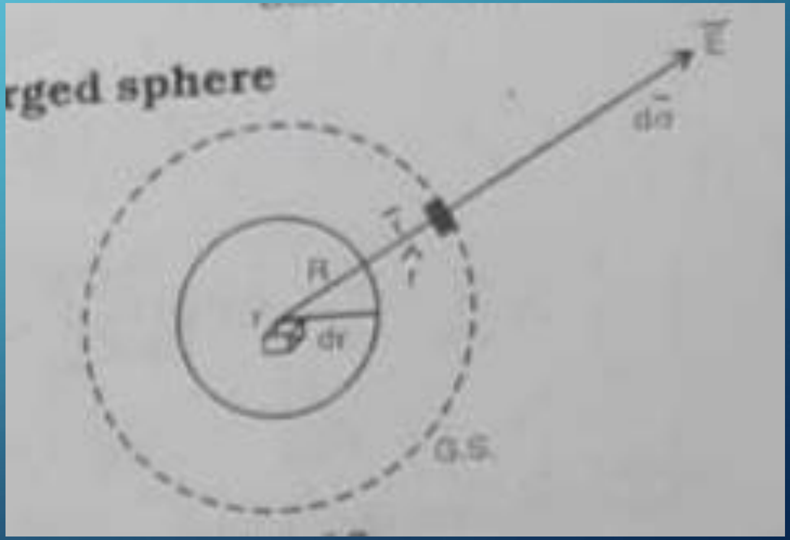
FIELD DUE TO NON UNIFORMLY CHARGED SPHERE

. The sphere is having non uniform charge distribution. Given that charge density ρ is

Proportional to distance from the centre of the sphere

$$\rho \propto r$$

$$\rho = kr$$



I) ***Field outside the sphere (r > R)1***

Let the field to be determined at a distance r which is outside the charge sphere.A Gaussian surface is to be constructed with radius ‘r’ which is concentric with the charged sphere of radius R.

Applying Gauss’s law,

$$\int_{G.S} E.da = \frac{Q_{enc}}{\varepsilon o} \qquad (1)$$

For a charged sphere. E and da are in same direction radially outwards

$$\int_{G.S} E.da = \int Eda \qquad (2)$$

$$E \int_{G.S} da = \frac{Q_{enc}}{\varepsilon o}$$

$$E\ _E 4\square r^2 = \frac{Q_{enc}}{\varepsilon o} \qquad (3)$$

$$Q_{enc} = \int_v \rho \, d\tau \qquad (4)$$

$$\rho = kr \qquad (5)$$

If the volume element $d\tau$ is at a distance r$'$ from the sphere then.

$$\rho = kr' \qquad (6)$$

In terms of SPC; $d\tau = r^2 \sin\theta\, d\theta\, dr\, d\phi$

As $d\tau$ *is at a distance* r',

$$d\tau = r^2 \sin\theta\, d\theta\, dr'\, d\phi \qquad (7)$$

$$Q_{enc} = \int_v kr'\, r'^2 \sin\theta\, d\theta\, dr'\, d\phi \qquad (8)$$

$$= \int_v kr'^3 \sin\theta\, d\theta\, dr'\, d\phi = k \int^R_0 r'^3\, dr' \int^{□}_0 \sin\theta\, d\theta \int^{2□}_0 d\phi$$

$$= k\, [r'^4]^R_0\, 4□ = k\ R^4\ 4□$$

$$4 \underline{\qquad\qquad} \qquad 4 \quad \underline{\qquad}$$

$$Q_{enc} = k□R^4 \qquad (9)$$

Sub for Q_{enc} in eqn (3)

$$E\,4\pi r^2 = \frac{Q_{enc}}{\varepsilon o} = \frac{k\pi R^4}{\varepsilon o}$$

$$E = \frac{Kr^4}{4\varepsilon o r^2}\,\acute{r} \qquad (10)$$

Eqn (10) gives the field outside the non uniformly charged sphere.

$$\boxed{E \propto \frac{1}{r^2}}$$

II) Field on the surface of the sphere

On the surface r = R

$$E = \frac{kR^4}{4\varepsilon o R^2} \acute{r} \qquad (11)$$

$$E = \frac{kR^2}{4\varepsilon o} \acute{r}$$

On the surface ,

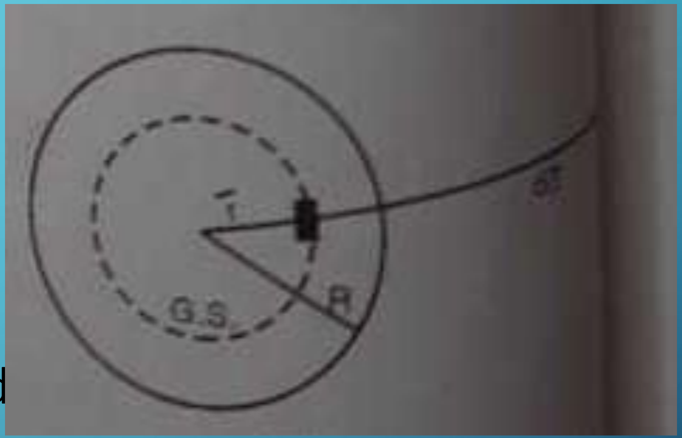
$$E \propto \boxed{R^2}$$

III) Field inside the sphere (r < R)

Let the field is to be determined at a distance r from the centre of the sphere, so Gaussian surface is to be constructed inside the sphere with radius ‘r’.

Applying gauss’s law.

$$\int_{G.S} E.da = \frac{Q_{enc}}{\varepsilon o} \qquad (12)$$



$$\int_{G.S} E.da = \frac{\int_v \rho dr}{\varepsilon o}$$

For charged sphere. E is radially outwards and parallel to d

$$E \int_{G.S} da = \frac{1}{\varepsilon o} \int_v kr' r'^2 \sin\theta \, d\theta \, dr' \, d\phi \qquad (13)$$

$$= \frac{1}{\varepsilon o} \int_v kr'^3 \sin\theta \, d\theta \, dr' \, d\phi$$

$$E4\pi r^2 = \frac{1}{\varepsilon o} \int_0^r kr'^3 \, dr' \int_0^\pi \sin\theta \, d\theta \int_0^{2\pi} d\phi$$

$$E4\pi r^2 = \frac{1}{\varepsilon o} \, k \, \frac{[\, r'^4 \,]_0^r \, 4\pi}{\varepsilon o}$$

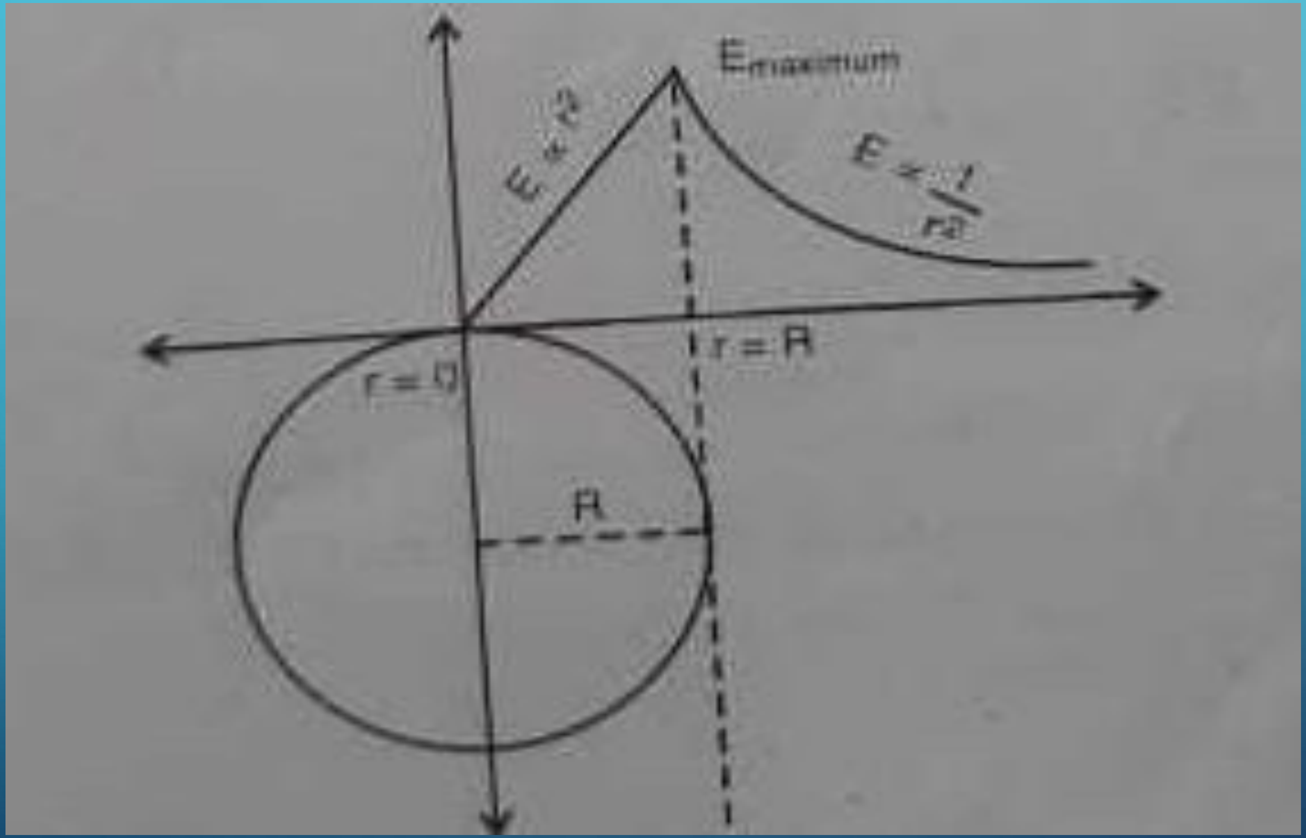
$$Er^2 = \frac{1}{\varepsilon o} \, \frac{k \, r^4}{\varepsilon o}$$

$$E = \frac{kr^2}{4\varepsilon o} \, \hat{r} \qquad (14)$$

Eqn (14) gives the field inside the non uniformly charged sphere.

$$\boxed{E \infty r^2}$$

IV) Graphical representation: E of Non Uniformly charged sphere.



FIELD DUE TO UNIFORMLY CHARGED CYLINDER

I) **Outside the cylinder (r > R)** Let the field is to be determined at a distance r from the axis of uniformly charged cylinder. Then guassian surface is to be constructed at a distance r outside the charged cylinder. which will be co – axial with the charged cylinder. Applying Gauss's law to the cylindrical guassian surface.

$$\int_{G.S} E.da = \frac{Q_{enc}}{\varepsilon o} \qquad (1)$$

$$\int_{G.S} E.da = 2 \int_{\text{end side (E.S)}} E.da + \int_{\text{curved side (C.S)}} E.da$$

For a long charged cylinder E is alwayz radially outward, so it can be taken that E and da are in same direction for the curve side and E and da at right angles for the end sides

eqn (2) is changes to,

$$\int_{G,S} E.da = 2 \int_{E.S} Eda \cos 90 + \int_{C.S} Eda \cos 0$$

$$\int_{G.S} E.da = \int_{C.S} E\, da$$

$$= E \int_{c.s} da$$

$\int_{G.\,S} E.da \;=\; E * (\text{ Total area of curved surface})$

$\int_{G.\,S} E.da = E\, 2\pi\, r\, L$

Where L is the length of the gaussian cylinder

Applying eqn (4) in (1)

$$E\, 2\pi\, r\, L = \frac{Q_{enc}}{\varepsilon o}$$

Q_{enc} = charge enclosed by the gaussian cylinder

= volume of charged cylinder enclosed by gaussian cylinder * ρ

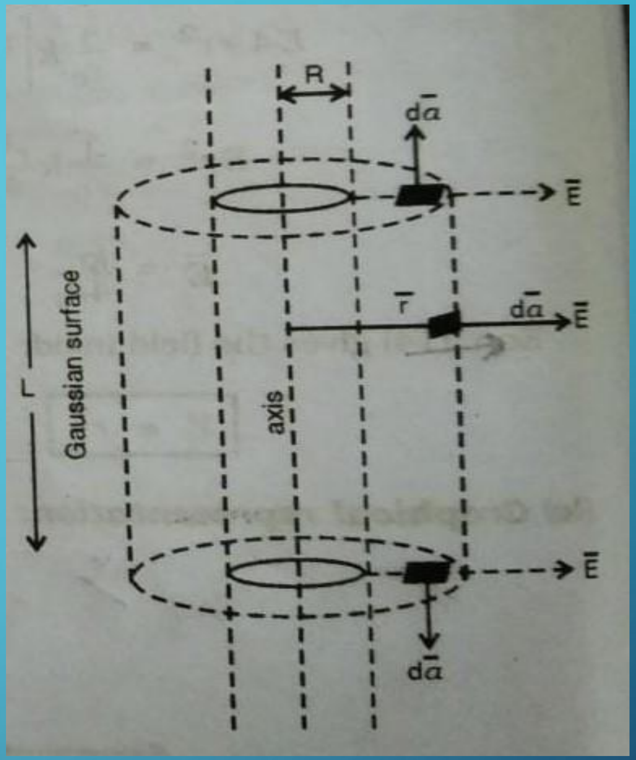
= $\pi R^2 L \rho$

$$E 2\pi r\, L = \frac{\pi R^2 L \rho}{\varepsilon o}$$

$$E = \frac{R^2 \rho}{2\varepsilon o r} \qquad r$$

Outside the cylinder,

$$\boxed{E \infty \frac{1}{r}}$$



II) Field on the surface

On the surface r = R

$$E = \frac{R^2 \rho}{2\varepsilon o R} \quad R$$

$$E = \frac{R\rho}{2\varepsilon o} \quad R$$

III) Field inside the cylinder (r < R)

Let field is to be determined at a distance r inside the cylinder, then gaussian surface is to be constructed inside the charged cylinder which is essentially co-axial to the charged cylinder

Applying the gauss's law guassian cylinder,

$$\int_{G.S} E.da = \frac{Q_{enc}}{\varepsilon o}$$

$$\int_{G.S} E.da = 2 \int_{E.S} E.da + \int_{C.S} E.da$$

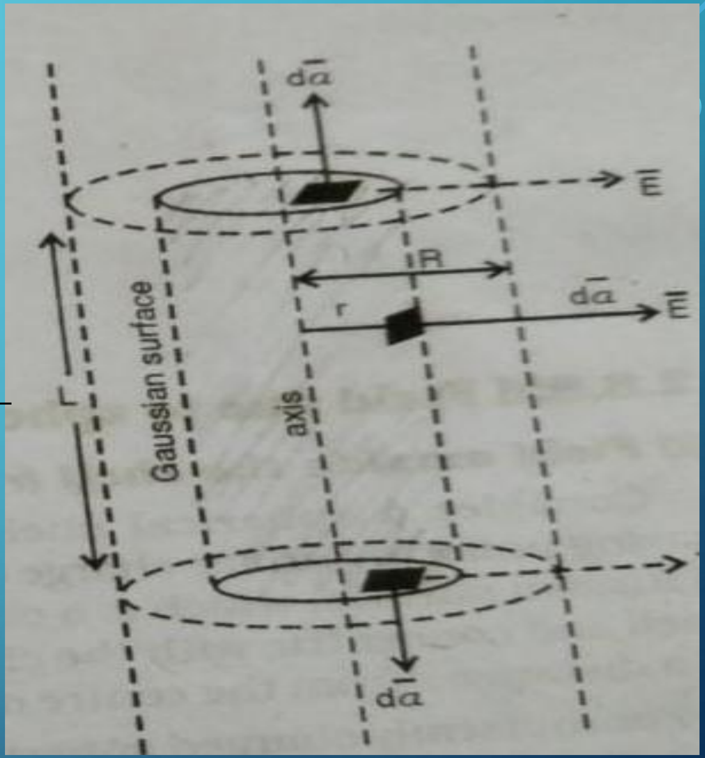
At the end E and da are perpendicular to each other; so flux through the ends zero

$$\int_{G.S} E.da = E \int_{C.S} da$$

$$= E \int_{C.S} da = E2 \square r L$$

Applying eqn (10) in eqn (7)

$$E2\square r L = \frac{\text{volume of charged cylinder inside G.S} * \rho}{\varepsilon o}$$



$$E2\square r L = \frac{\square r^2 L \rho}{\varepsilon o}$$

$$E = \frac{r\rho}{2\varepsilon o} \acute{r}$$

Inside the cylinder

$$\boxed{E \infty r}$$

IV) Graphical representation of field due to charged cylinder