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Probability Distributions

Module 1

- STANDARD DISTRIBUTIONS

In this module some important discrete distributions and continuous distributions are discussed

Discrete distributions

- Discrete distributions discussed in this module are the following:
 1. Bernoulli
 2. Binomial
 3. Poisson
 4. Geometric
 5. Negative Binomial
 6. Uniform

Bernoulli Distribution

Definition

A random variable X which takes two values 0 and 1 with probabilities q and p respectively ie, $P(X=1)=p$, $P(X=0)=q$, $q=1-p$ is called a Bernoulli variate and is said to have a Bernoulli distribution.

In another way we can say:

A random variable X is said to follow Bernoulli distribution if its pdf is given by

$$f(x) = p^x (1 - p)^{1-x}, x = 0, 1$$

= 0 elsewhere

Examples

1. An unbiased die is thrown once. Occurrence of 6 is considered as a success. Let X denote the number of successes. Here X takes the values 0 with probability $\frac{5}{6}$ and takes the value 1 with probability $\frac{1}{6}$.

2. An unbiased die is thrown once. Occurrence of 5 or 6 is considered as a success. Let X denote the number of successes. Here X takes the values 0 with probability $\frac{2}{3}$ and takes the value 1 with probability $\frac{1}{3}$.

3. An unbiased coin when tossed shows head or tail. If the outcome is head call it as success . Then $X=0$ or 1 according as outcome is tail or head .Here $P(X=0)=P(X=1)=\frac{1}{2}$

4. An unbiased die is thrown once. Occurrence of an even number is considered as a success. Let X denote the number of successes . Here X takes the values 0 and 1 with probability $\frac{1}{2}$

Moments of Bernoulli distribution

The r^{th} moment about origin is

$$\mu_r' = E(X^r) = 0 \cdot q + 1 \cdot p = p, \quad r=1,2,\dots$$

- Mean $=E(X)=p$
- Variance $=E(X^2)-\{E(X)\}^2$
 $=p-p^2=p(1-p)$
 $=pq.$

Moment Generating Function

- $Mgf = E(e^{tX})$
 $= e^0 \cdot q + e^t \cdot p$
 $= q + pe^t.$

Problem

- $X=0$ or 1 according as an unbiased coin when tossed shows head or tail. Find the mgf.

Answer:

Pdf is given by

$$X \quad 0 \quad 1$$

$$P_x \quad \frac{1}{2} \quad \frac{1}{2}$$

$$\text{mgf is } q+pe^t = \frac{1}{2} + \frac{1}{2} e^t.$$

Assignment

- $X=0$ or 1 according as an unbiased coin when tossed shows head or tail. Find the first four central moments.