TIME INDEPENDENT SHRODINGER EQUATION

By

Haisel Mathew

Dept. of Physics

In many situations, potential energy of a particle does not depend on time.In this case ,the wave function is given by

$$\psi_{(x,t)} = Ae^{-\frac{i}{\eta}(Et - Px)} = Ae^{-\frac{ipx}{\eta}} e^{\frac{-iEt}{\eta}}$$

$$\psi_x = Ae^{-\frac{ipx}{\eta}} \textit{ is a position dependent function}$$

$$\psi_{(x,t)} = \psi_x e^{\frac{-iEt}{\eta}} \quad ......(1)$$

Differentiating Equ .(1) w.r.to time , we get

$$\frac{\partial \psi}{\partial t} = \psi_x e^{\frac{-iEt}{\eta}} \left(\frac{-iE}{\eta} \right) ...(2)$$

$$\psi_{(x,t)} = Ae^{-\frac{i}{\eta}(Et-Px)} = \psi_x e^{\frac{-iEt}{\eta}} \ldots\ldots(1)$$

$$Differentiating\ Equ.(1)\ w.r.to\ x\ twice,$$

$$we\ get,\ \frac{\partial^2\psi_{(x,t)}}{\partial x^2} = e^{\frac{-iEt}{\eta}} \frac{\partial^2\psi_x}{\partial x^2} \ldots(3)$$

The time dependent schrodinger equ.is given by

$$i\eta\frac{\partial\psi}{\partial t}=\frac{-\eta^2}{2m}\frac{\partial^2\psi}{\partial x^2}+U\psi$$

$$substitute\ \ for\ \ \frac{\partial\psi}{\partial t}\ from$$

$$the\ \ equation(2), weget$$

$$i\eta\psi_x e^{\frac{-iEt}{\eta}}\left(\frac{-iE}{\eta}\right)=\frac{-\eta^2}{2m}\frac{\partial^2\psi}{\partial x^2}+U\psi$$

$$sustitute\ \ for\ \frac{\partial^2\psi}{\partial x^2}\ from\ equ.(3)$$

$$i\eta\psi_x e^{\frac{-iEt}{\eta}}\left(\frac{-iE}{\eta}\right) = \frac{-\eta^2}{2m} e^{\frac{-iEt}{\eta}} \frac{\partial^2\psi_x}{\partial x^2} + U\psi_x e^{\frac{-iEt}{\eta}}$$

$$Divide\ both\ sides\ by\ e^{\frac{-iEt}{\eta}}, we\ get$$

$$E\psi_x = \frac{-\eta^2}{2m}\frac{\partial^2\psi_x}{\partial x^2} + U\psi_{(x)}$$

$$E\psi_x - U\psi_{(x)} = \frac{-\eta^2}{2m}\frac{\partial^2\psi_{(x)}}{\partial x^2}$$

$$[E-U]\psi_x = \frac{-\eta^2}{2m}\frac{\partial^2\psi_{(x)}}{\partial x^2}$$

$$\frac{2m}{\eta^2}[E-U]\psi_x = -\frac{\partial^2\psi_{(x)}}{\partial x^2}$$

$$\frac{\partial^2\psi_{(x)}}{\partial x^2} + \frac{2m}{\eta^2}[E-U]\psi_x = 0 \ldots\ldots(4)$$

This is the one dim *ensional time independen t Schrodinge r equation .*

The three dimensional time independent Schrodinger equation is given by

$$\frac{\partial^2 \psi_{(x)}}{\partial x^2} + \frac{\partial^2 \psi_{(x)}}{\partial y^2} + \frac{\partial^2 \psi_{(x)}}{\partial z^2} + \frac{2m}{\eta^2}\left[E - U\right]\psi_x = 0.$$

$$ie\,\nabla^2 \psi_{(x)} + \frac{2m}{\eta^2}\left[E - U\right]\psi_x = 0.$$

THANK YOU