WORK AND ENERGY

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Work

- While most people feel that work is done when you "work on a problem" or "do homework," physicists say work has only been done when a force is applied to an object and the object moves in the direction of the applied force.

Definition of Work, *W* (force in the direction of displacement)

$$\text{work} = \text{force} \times \text{distance}$$
$$W = Fd$$

SI unit: newton-meter $(\mathrm{N \cdot m}) = \text{joule } (\mathrm{J})$

Work and Energy

- As the equation $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$ indicates, the total work done on an object equals the change in its kinetic energy. This connection is known as the work-energy theorem:

Work-Energy Theorem

$$\text{total work} = \text{change in kinetic energy}$$

$$W_{\text{total}} = \Delta KE$$

$$= \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

Work and Energy

- The following example shows how work is related to the change in kinetic energy.

How much work is required for a 74-kg sprinter to accelerate from rest to a speed of 2.2 m/s?

Solution

Since the initial speed is zero, $v_i = 0$, we have

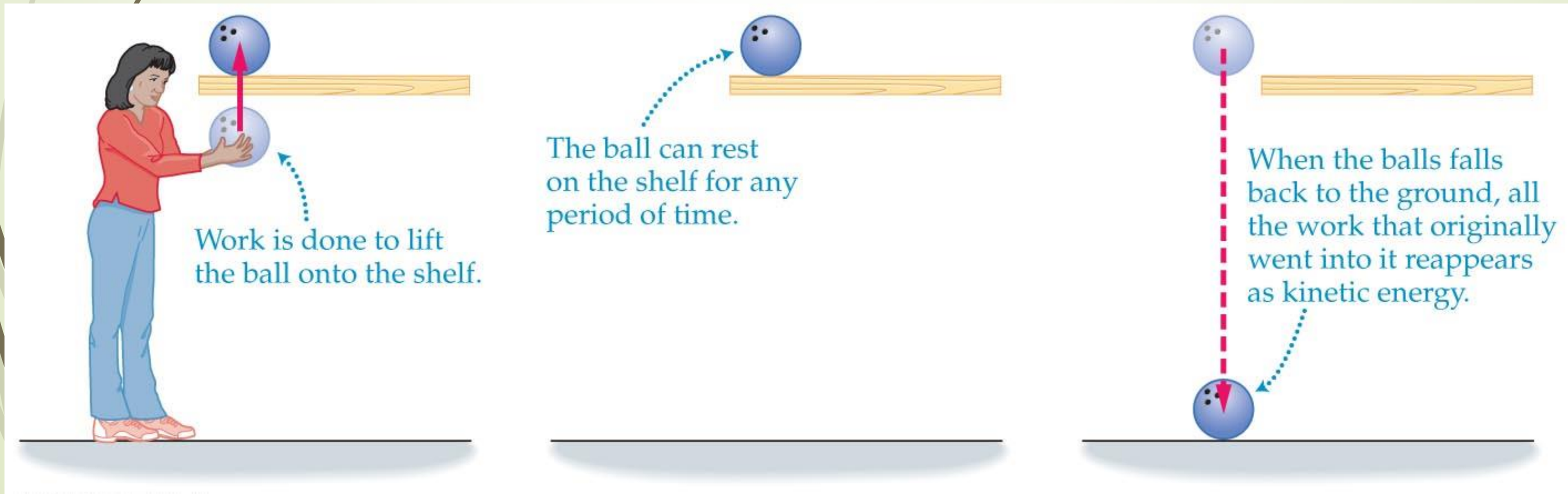
$$\begin{aligned} W_{total} &= \Delta KE = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 \\ &= \tfrac{1}{2}(74\ \text{kg})(2.2\ \text{m/s})^2 - \tfrac{1}{2}(74\ \text{kg})(0\ \text{m/s})^2 \\ &= \boxed{180\ \text{J}} \end{aligned}$$

Work and Energy

- The sign of the work is related to the change in kinetic energy:
  - If the total work is positive, then the kinetic energy increases.
  - If the total work is negative, then the kinetic energy decreases.
  - If the total work is zero, then there is no change in kinetic energy.

Work and Energy

- As the figure below indicates, work must be done to lift a bowling ball from the floor onto a shelf.
- Even though the ball has no kinetic energy once it's resting on the shelf, the work done in lifting the ball is not lost—it is stored as potential energy.



Work and Energy

- Energy that is stored for later use is referred to as potential energy, or *PE*.
- Potential energy has several forms, one of which is gravitational potential energy.
- The gravitational potential energy equals the work required to lift an object to a given height.
- Lifting a mass m from the ground to a height h requires a force mg. Thus the work done, and the potential energy acquired, equals force times distance, or

$$W = mgh$$

Definition of Gravitational Potential Energy, $PE_{gravity}$

$$\text{potential energy of gravity} = \text{mass} \times \begin{pmatrix}\text{acceleration due} \\ \text{to gravity}\end{pmatrix} \times \text{height}$$

$$PE_{gravity} = mgh$$

SI unit: $kg \cdot m^2/s^2 = J$

- The following example shows how the gravitational energy is calculated.

Find the gravitational potential energy of a 65-kg person sitting on a diving board that is 3.0 m high.

Solution

Substituting $m = 65$ kg and $h = 3.0$ m in $PE_{gravity} = mgh$ yields

$$\begin{aligned} PE_{gravity} &= mgh \\ &= (65\ \text{kg})(9.81\ \cancel{\text{m}}/\text{s}^2)(3.0\ \cancel{\text{m}}) \\ &= \boxed{1900\ \text{J}} \end{aligned}$$

Work and Energy

- Objects like rubber bands and springs that return to their original size and shape after being distorted are said to be elastic.
- Stretching a spring requires work. This work is stored in the stretched spring in the form of potential energy.
- The potential energy stored in a distorted elastic material is referred to as elastic potential energy.

Work and Energy

- When a spring is stretched by a distance *x*, the force exerted on the spring increases uniformly from 0 to *kx*, where *k* is the spring constant.
- Thus, the average force is exerted on the spring is $\frac{1}{2}kx$
- Since the average force is $\frac{1}{2}kx$ the work done in changing the length of the spring is the average force times the distance, or
- This work is stored as elastic potential energy.

$$W = \left(\tfrac{1}{2}kx\right)(x) = \tfrac{1}{2}kx^2$$

Definition of Spring Potential Energy, PE_{spring}

$$\text{potential energy of a spring} = \frac{1}{2} \times \left(\text{spring constant}\right) \times \left(\text{distance of stretch or compression}\right)^2$$

$$PE_{spring} = \tfrac{1}{2}kx^2$$

SI unit: $kg \cdot m^2/s^2 = J$

- The following example shows how elastic potential energy is calculated.

A spring with a spring constant of 120 N/m is compressed a distance of 2.3 cm. How much potential energy is stored in the spring?

Solution

Substituting $k = 120\ N/m$ and $x = 2.3\ cm = 0.023\ m$ in $PE_{spring} = \tfrac{1}{2}kx^2$ yields

$$\begin{aligned} PE_{spring} &= \tfrac{1}{2}kx^2 \\ &= \tfrac{1}{2}(120\ N/m)(0.023\ m)^2 \\ &= \boxed{0.032\ J} \end{aligned}$$

Conservation of Energy

- Energy takes many forms: mechanical, electrical, thermal, and nuclear.
- Any time work is done, energy is transformed from one form to another.
- One process might transform some kinetic energy into electrical potential energy; another might transform some spring potential energy into kinetic energy.
- However, no matter what the process, the total amount of energy in the universe remains the same. This is what is meant by the conservation of energy.
- To say that energy is conserved means that energy can never be created or destroyed—it can only be transformed from one form to another.

Conservation of Energy

- When frictional forces act on a system, such as when a car's brakes are applied, kinetic energy is transformed into thermal energy.
- In situations where all forms of friction can be ignored, no potential or kinetic energy is transformed into thermal energy. In this ideal case, the sum of the kinetic and potential energies is always the same.
- The sum of the kinetic and potential energies of an object is referred to as its mechanical energy. Thus,
  mechanical energy = potential energy + kinetic energy

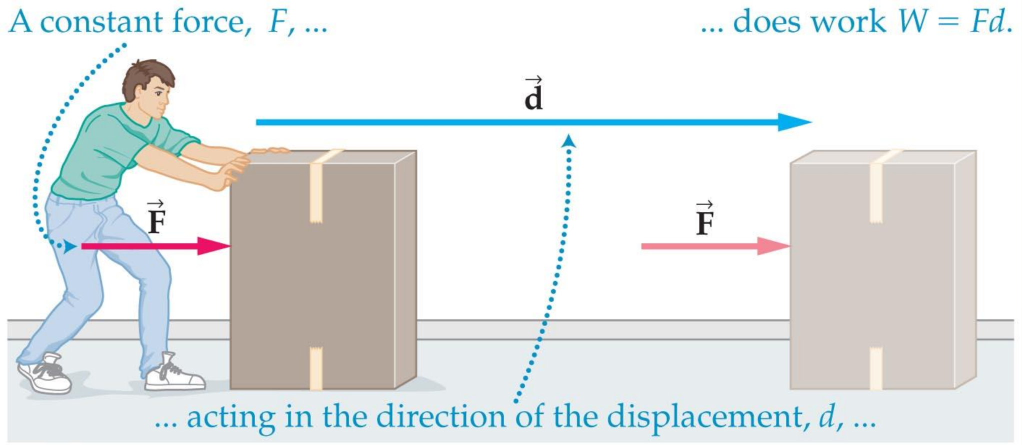
  $$E = PE + KE$$

- This means that mechanical energy is conserved.

THANKYOU

Work

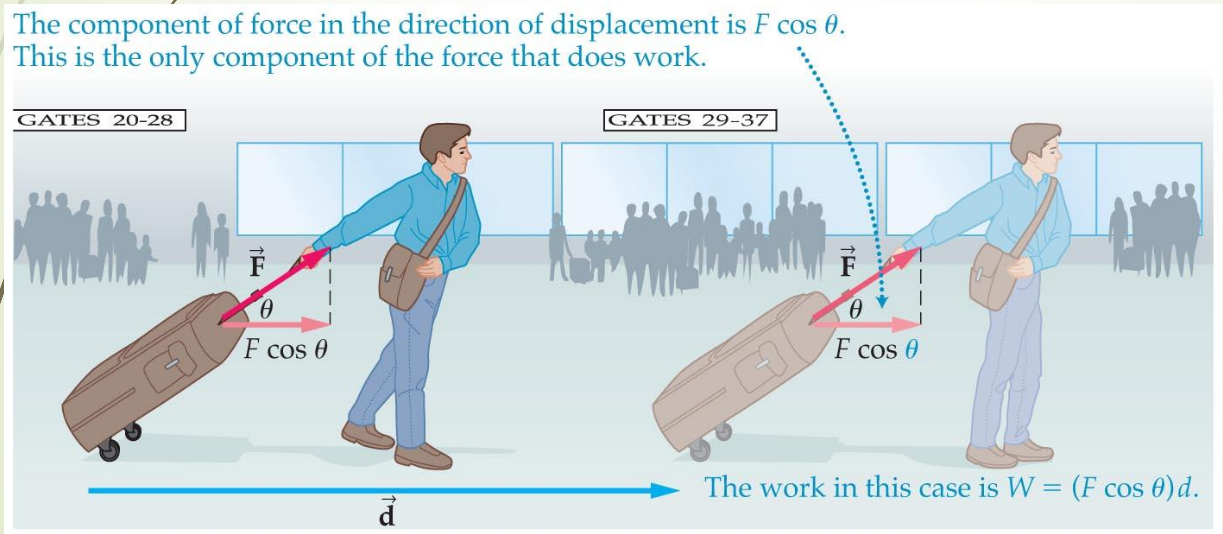
- The figure below illustrates work being done as a force pushes a box through a displacement. The work done equals *W* = *Fd*.



- The dimensions of work are force (newtons times distance (meters). The product of the two, N·m, is called the joule, in honor of physicist James Prescott Joule.

Work

- Work is easily calculated when the force and displacement are in the same direction, but how is work calculated when the force is at an angle to the displacement?
- The figure below shows a person pulling a suitcase at an angle θ with respect to the direction of motion.



Work

- In a case such as this, only the component of the force in the direction of the displacement does work.
- Notice in the previous figure that the component of force in the direction of displacement is $F\cos\theta$. Therefore, the work equals $Fd\cos\theta$.

Definition of Work, *W* (with force and displacement at an angle θ)

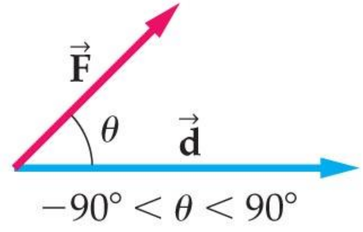
$$\text{work} = \text{force} \times \text{distance} \times \begin{pmatrix}\text{cosine of angle between} \\ \text{force and displacement}\end{pmatrix}$$

$$W = Fd\cos\theta$$

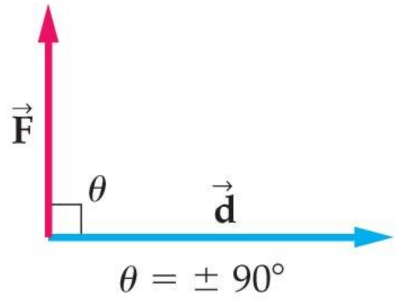
SI unit: joule (J)

Work

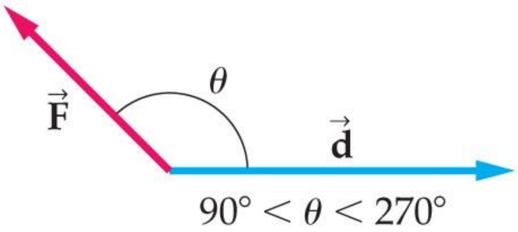
- Work can be positive, negative, or zero.
  - Work is positive if the force has a component in the direction of motion (Figure a).
  - Work is zero if the force has no component in the direction of motion (Figure b).
  - Work is negative if the force has a component opposite the direction of motion (Figure c).



(a) Positive work



(b) Zero work



(c) Negative work

Work

- When more than one force acts on an object, the total work is the sum of the work done by each force separately.
- For example, if $\vec{\mathbf{F}}_1$ does work W_1, force $\vec{\mathbf{F}}_2$ does work W_2, force $\vec{\mathbf{F}}_3$ does work W_3, and so on, the total work equals

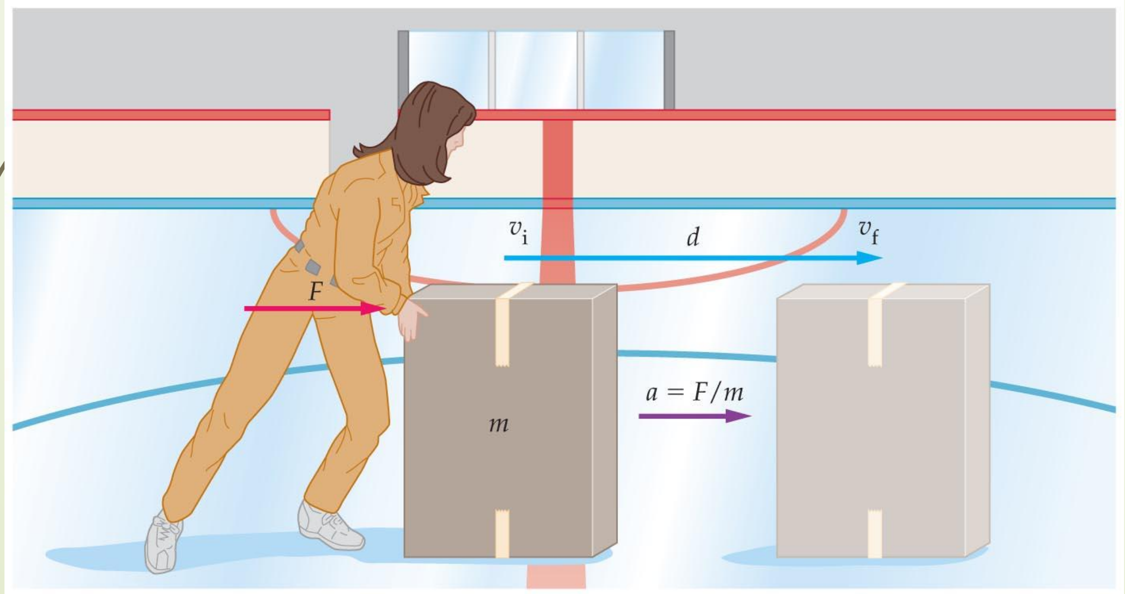
$$W_{total} = W_1 + W_2 + W_3 + \ldots$$

Work and Energy

- When work is done on an object, the object's energy changes. For example:
  - When you climb a mountain, your work goes into increasing your potential energy.
- Thus kinetic energy is energy of motion; potential energy is the energy of position or condition.

Work and Energy

- Newton's laws and the equations of motion may be used to derive a relationship between work and energy.
- In the figure below, a box is pushed across an ice-skating rink with a force *F*. Let's see how this force changes the box's energy.



Work and Energy

Kinetic energy is the energy of motion

We can use our knowledge of Newton's laws and the equations of motion to write a relationship between work and energy. Suppose you push a box

acceleration of the box is given by Newton's second law:

$$a = \frac{F}{m}$$

As you learned in Chapter 3, the speed and displacement of an accelerat
object are related by the equation of motion:

$$v_f^2 = v_i^2 + 2ad$$

A slight rearrangement gives

$$2ad = v_f^2 - v_i^2$$

Replacing a with F/m and multiplying both sides of the equation by $m/$
gives

$$Fd = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

Recalling from the previous lesson that work is $W = Fd$, we have

$$W = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

Work and Energy

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

- From the relationship $\frac{1}{2}mv^2$, we see that the work done on the box (or on any other object) is related to the quantity .
- The quantity $\frac{1}{2}mv^2$ is defined as the kinetic energy, or KE, of an object of mass *m* and speed v.

Definition of Kinetic Energy, *KE*

$$\text{kinetic energy} = \frac{1}{2}(\text{mass}) \times (\text{velocity})^2$$

$$KE = \frac{1}{2}mv^2$$

SI unit: $\text{kg} \cdot \text{m}^2/\text{s}^2 = \text{J}$

Work and Energy

- In general, the kinetic energy of an object is the energy due to its motion.
- Kinetic energy is measured with the joule, the same unit used to measure work.

Work and Energy

- The kinetic energy increases linearly with the mass and with the square of the velocity, as the following example indicates.

A 3900-kg truck is moving at 6.0 m/s. **(a)** What is its kinetic energy? **(b)** What is the truck's kinetic energy if its speed is doubled to 12 m/s?

Solution

(a) Substituting $m = 3900$ kg and $v = 6.0$ m/s into $KE = \frac{1}{2}mv^2$ gives

$$\begin{aligned} KE &= \tfrac{1}{2}mv^2 \\ &= \tfrac{1}{2}(3900\ \text{kg})(6.0\ \text{m/s})^2 \\ &= \boxed{70{,}000\ \text{J}} \end{aligned}$$

(b) Kinetic energy depends on the speed squared. Therefore, doubling the speed quadruples the kinetic energy.

$$KE = 4(70{,}000\ \text{J}) = \boxed{280{,}000\ \text{J}}$$