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Electric potential
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ELECTRIC POTENTIAL(V)

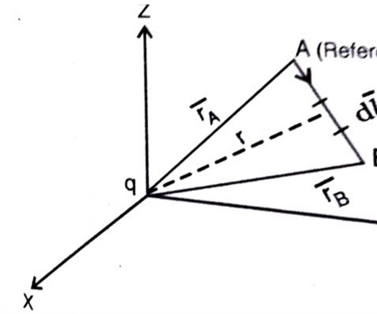
Electric potential (V) is defined as the work done in moving a unit positive charge from a reference point to a point in an electric field.

ELECTRIC FIELD POTENTIAL DUE TO POINT CHARGE

In the case of electric field the reference point is taken as infinity.

Field produced by a charge q at a distance r is,

$$\bar{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$$



When a unit positive charge is moved from A to B as shown in the figure, work is to be done against the field.

The small work to be done to move unit positive charge for a very small displacement dl,

dw = F.dl

F = -E, which is the force applied to counter the field.

Total work to be done in moving unit positive charge from a reference point to a given point in the field,

$$\int_A^B dw$$
$$\int_A^B -\bar{E}\cdot d\bar{l}$$
$$\int_A^B \bar{E}\cdot d\bar{l}$$

shows that work done can be obtained as the negative line integral of the electric field. As per the ition of electric potential, V,

$$\mathrm{V}=\mathrm{w}=-\int_{A=\infty}^{B=(\text{ point m}\,\bar{E})} \bar{E}\cdot d\bar{l}$$

$$d\bar{l}=dr\hat{r}+rd\theta\dot{\theta}+r\sin\theta d\phi\dot{\phi}$$

$$\bar{E}\cdot d\bar{l}=\frac{q}{4\pi\varepsilon_0 r^2}\hat{r}\cdot(dr\bar{r}+rd\theta\hat{\theta}+r\sin\theta d\phi\hat{\phi})$$

$$=\frac{q}{4\pi\varepsilon_0 r^2}dr$$

$$\therefore\quad V=-\int_{\infty}^{r=r_B}\frac{q}{4\pi\varepsilon_0 r^2}dr$$

$$V=\frac{-q}{4\pi\varepsilon_0}\int_{\infty}^{r_B}\frac{dr}{r^2}$$

$$=\frac{-q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{\infty}^{T_B}=\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_B}-\frac{1}{\infty}\right]$$

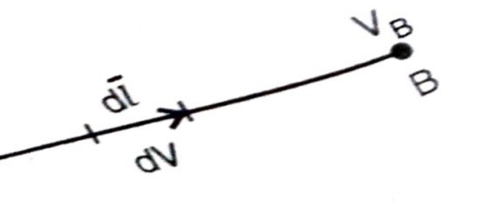
$$\mathrm{V}=\frac{q}{4\pi\varepsilon_0 r_B} \qquad \text{-----(7)}$$

Eqn. (7) gives the expression for potential at the point B with position vector $\mathrm{r_B}$. In general,

$$\mathrm{V}=\frac{q}{4\pi\varepsilon_0 r}$$

POTENTIAL DIFFERENCE

tial difference is defined as the work done on a unit positive charge between two p
unit positive charge is moved from A to B, then potential difference = VB-VA, whe
/A are the electric potential at the point B and A respectively. Here VB >VA since w
on the unit positive charge to move from A to B. If dv represents a very small pote
difference along the path connecting A and then,



$$V_B - V_A = \int_A^B dV$$

$$dV = -\bar{E} \cdot d\bar{l}$$

$$\therefore V_B - V_A = -\int_A^B \bar{E} \cdot d\bar{l}$$

$$= -\int_{r_A}^{r_B} \frac{q}{4\pi\varepsilon_0 r^2} \hat{r} \cdot (dr\hat{r} + rd\theta\dot{\theta} + r\sin\theta d\phi\dot{\phi})$$

$$= -\int_{r_A}^{r_g} \frac{q}{4\pi\varepsilon_0 r^2} dr$$

$$= -\frac{q}{4\pi\varepsilon_0} \int_{r_A}^{r_B} \frac{dr}{r^2} = -\frac{q}{4\pi\varepsilon_0} \left[-\frac{1}{r}\right]_{T_A}^{r_B}$$

$$V_B - V_A = \frac{q}{4\pi\varepsilon_0} \left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

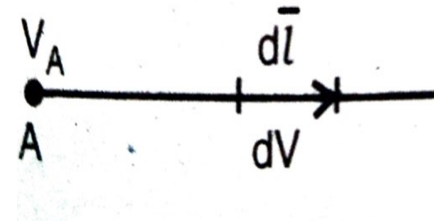
ELECTRIC FIELD AS THE NEGATIVE GRADIENT OF POTENTIAL

represents a small p.d. between two close points in un electric field of ity $\bar{E}$. If $\bar{d}\,\bar{l}$ represents the distance between two points, then,

$$\mathrm{dV} = \mathrm{dw} = -\bar{E} \cdot d\bar{l}$$

ematically dV can be considered as the total differentlal of V.

n SPC system. $\mathrm{dV} = \frac{\partial V}{\partial r} dr + \frac{\partial V}{\partial \theta} d\theta + \frac{\partial V}{\partial \phi} d\phi$



be proved that r.h.s. of eqn. (2)

$$= \nabla V \cdot d\bar{l}$$

e $d\bar{l}$ is the elemental displacement in SPC.

$$\mathrm{Ie., \ dV} = \nabla V \cdot d\bar{l}$$

omparing eqn. (1) and eqn. (2)

$$-\bar{E} \cdot d\bar{l} = \nabla V \cdot d\bar{l}$$

$$-\bar{E} = \nabla V$$

$$\therefore \quad \bar{E} = -\nabla V$$

TENTIAL DUE TO INFINITELY LONG WIRE

V be the distance r from an infinitely long line charge distribution having linear charge density λCm-1
ential is given by,

$$\mathrm{V}=-\int_{\text{ref point}}^{\text{point } \ln E} \cdot dl$$

$$\bar{E}=\frac{\lambda}{2\pi\varepsilon_0 r}\dot{r}$$

$$d\bar{l}=dr\bar{r}+rd\theta\dot{\theta}+r\sin\theta d\phi\dot{\phi}$$

and reference point is given by", $r=a$

$$\therefore \mathrm{V}=\int_{r=a}^{r=r}\frac{\lambda}{2\pi\varepsilon_0\mathrm{r}}\hat{r}\cdot(dr\bar{r}+rd\theta\hat{\theta}+r\sin\theta d\phi\hat{\phi})$$

$$=-\int_a^r\frac{\lambda}{2\pi\varepsilon_0 r}dr$$

$$=\frac{-\lambda}{2\pi\varepsilon_0}\int_a^r\frac{dr}{r}$$

$$=\frac{-\lambda}{2\pi\varepsilon_0}[\log r]_a^r=\frac{-\lambda}{2\pi\varepsilon_0}[\log r-\log a]$$

$$\mathrm{V}=\frac{1}{2\pi\varepsilon_0}[\log a-\log \mathrm{r}]$$

TENTIAL V OUTSIDE THE SHELL(r>R)

$$V_{out} = -\int_{r=\infty}^{r=r} \overline{E}_{out} \cdot d\overline{l}$$

$$V_{out} = -\int_{\infty}^{r} \frac{q}{4\pi\varepsilon_0 r^2}\hat{r} \cdot (dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta d\phi\dot{\phi})$$

$$V_{out} = -\int_{\infty}^{r} \frac{q}{4\pi\varepsilon_0 r^2} dr$$

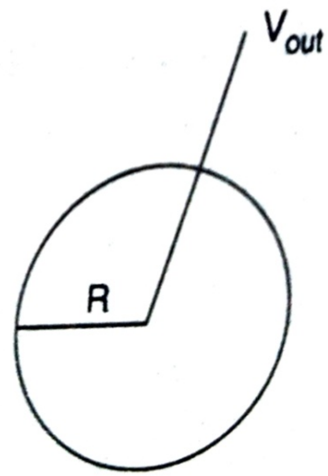
$$V_{out} = -\frac{q}{4\pi\varepsilon_0}\int_{-\infty}^{r} \frac{1}{r^2} dr$$

$$= -\frac{q}{4\pi\varepsilon_0}\left[\frac{-1}{r}\right]_{\infty}^{r}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$V_{out} = \frac{q}{4\pi\varepsilon_0 r}$$

$$q = 4\pi R^2\sigma$$



here R , radius of the shell and σ surface charge density.

$$V_{out} = \frac{4\pi R^2\sigma}{4\pi\varepsilon_0 r}; V_{out} = \frac{R^2\sigma}{\varepsilon_0 r}$$

II)POTENTIAL ON THE SURFACE OF THE SHELL(r=R)

$$\mathrm{V}_{\text{surface}} = -\int_{r=\infty}^{r=R} \bar{E}_{\text{out}} \cdot d\bar{l}$$

$$= -\int_{\infty}^{R} \frac{q}{4\pi\varepsilon_0 r^2}\hat{r} \cdot (dr\dot{r} + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi})$$

$$= -\int_{\infty}^{R} \frac{q}{4\pi\varepsilon_0 r^2} dr$$

$$V_{\text{surface}} = \frac{-q}{4, n\varepsilon_0}\int_{\infty}^{R} \frac{1}{r^2} dr$$

$$= -\frac{q}{4\pi\varepsilon_0}\left[\frac{-1}{r}\right]_{\infty}^{R}$$

$$= \frac{q}{4.\,\pi\varepsilon_0}\left[\frac{1}{r}\right]_{\infty}^{R} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\infty}\right]$$

$$\mathrm{V}_{\text{surface}} = \frac{q}{4\pi\varepsilon_0 R}$$

$$\mathrm{q} = 4\pi\mathrm{R}^2\sigma$$

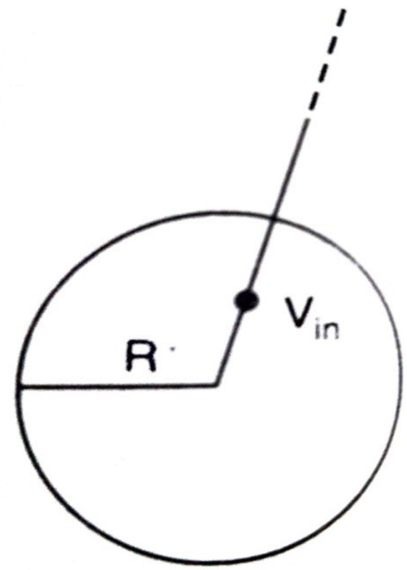
$$\mathrm{V}_{\text{surface}} = \frac{\mathrm{R}\sigma}{\varepsilon_0}$$

II)POTENTIAL INSIDE THE SHELL(r<R)

$= r$ (inside)

$$= -\int_{r=\infty} \bar{E} \cdot d\bar{l}$$

$$V_{\text{in}} = -\left[\int_{r=\infty}^{r=R} \bar{E}_{\text{out}} \cdot d\bar{l} + \int_{r=R}^{r=\text{ (inside)}} \bar{E}_{\text{th}} \cdot d\bar{l}\right]$$



$$\bar{E}_{in} = \text{ electric field inside the shell } = 0$$

$$\mathrm{V}_{\text{in}} = -\int_{\infty}^{R} \bar{E}_{\text{out}} \cdot d\bar{\mathrm{l}}$$

$$\mathrm{V}_{\text{in}} = \frac{\mathrm{q}}{4\pi\varepsilon_0 \mathrm{R}} = \frac{\mathrm{R}\sigma}{\varepsilon_0}$$

$$\mathrm{V}_{\text{in}} = \mathrm{V}_{\text{surface}}$$

Inside the charged shell potential is constant and equal to the surface of the shell

POISSON'S EQUATION

rential form of gauss's law is given by

$$\nabla \cdot \bar{E} = \frac{\rho}{\varepsilon_0}$$

$$\bar{E} = -\nabla V$$

$$\therefore \nabla \cdot (-\nabla V) = \frac{\rho}{\varepsilon_0}$$

$$-\nabla^2 V = \frac{\rho}{\varepsilon_0}$$

$$\nabla^2 V = \frac{-\rho}{\varepsilon_0}$$

is called Poisson's equation.

When $\rho = 0$

$$\nabla^2 V = 0$$

is called Laplace's equation.

THANK YOU