INTRODUCTION TO MEASURE THEORY

ALPHY JOSE

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DEFINITION

Definition

Measure is a set function satisfying the following properties

1. Measure of an interval is its length
2. Measure is translation invariant
3. Measure is countably additive over countable disjoint unions of sets

Outer measure

Outer measure

For a set A of real numbers, consider the countable collections

$\{I_k\}$ of non empty open bounded intervals covering A. The

outer measure of A, $m^*(A)$ is defined by

$m^*(A) = inf\{\sum_{k=1}^{\infty} l(I_k) \mid A \subseteq \cup I_k\}$

Properties of Outer measure

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* Outer measure is defined for all sets of real numbers

* The outer measure of an interval is its length

* Outer measure is translation invariant

* Outer measure is countably sub additive over any countable

collection of sets, disjoint or not

Measurable sets

Definition

A set E is said to be **measurable** provided for any set A,

$$m^*(A) = m^*(A \cap E) + m^*(A \cap E^C)$$

Measurable sets

Properties Measurable sets

1. The empty set is measurable
2. The set $\mathbb{R}$ is measurable
3. A set is measurable if and only if its compliment is measurable
4. Any set of outer measure zero is measurable. In particular, any countable set is measurable.

Measurable sets

Properties Measurable sets

1. The union of a finite collection of measurable sets is measurable.
2. The union of a countable collection of measurable sets is measurable.

σ Algebra

Definition

A collection of subsets of is called a σ **Algebra** if it is closed with respect of the formation of complements and countable unions.

By De-Morgan's Law such a collection will be closed with respect to the formation of countable intersections.

Remarks

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1. Every interval is measurable
2. The translate of a measurable set is measurable.

σ Algebra of Measurable sets

σ Algebra of Measurable sets

By the properties of measurable sets, the collection of all measurable sets in $\mathbb{R}$ forms a σ Algebra.

Also it will contain all the Borel sets in , ie, Each interval, each open set, each closed set, each G_δ set and each F_σ set is measurable.

HAVE A NICE DAY