SEQUENCE , SERIES AND PROGRESSION

SUBJECT: BASIC NUMERICAL SKILLS

STEFY M M

DEPT OF COMMERCE

ACADEMIC YEAR :2020-2021

ARITHMETIC PROGRESSION

IMPORTANT POINTS

- **Nth term**

  a+(n-1)d

  a : First term

  n : Position of the term

  d : common difference [d = a2 – a1]

- **General term**

  a , a+d , a+2d ,a+3d.....

SUM OF N TERMS OF AN AP

- $S_n = n/2 [2a + (n-1)d]$

  when last term is not given

- $S_n = n/2$ [first term + Last term]

  When last term is given

1. Find the sum of 20 items of the series 3 5 7 9

A. A= 3 d= 5-7=2 n=20

Sn = n/2[2a+(n-1)d]

= 20/2 [2×3+(20-1)2]

= 10 [6+19×2]

= 10×44

= 440

2 . Find the sum of the series where 1st term is 5 and 15th term is -23 .

A. A=5 n= 15 Ln= -23

Sn =n/2[first term + last term]

= 15/2 [5 + (-23)]

= 7.5 × -18

=-135

ARITHMETIC MEAN (AM)

- If a b c are in an AP then b is said to be the AM between a and c

$$AM = (a+b) / 2$$

Eg : Find AM between 5 and 8

$$AM = (5+8)/2$$

$$= 13/2 = 6.5$$

ie, 6.5 is the AM between 5 and 8

IF SUM IS GIVEN

- 3 numbers in AP can be assumed as a-d , a , a+d
- 4 numbers in AP can be assumed as a-3d , a-d , a+d , a+ 3d

61. Find 3 numbers in AP where Sum is 9 and the product is -165

A. 3 numbers are $a-d$, a, $a+d$

$$S_n = (a-d) + a + (a+d) = 9$$

$$3a = 9$$

$$a = 9/3 = \underline{3}$$

Substitute 3 in equation (2).

$$(a-d) \times a \times (a+d) = -165$$

$$(3-d) \times 3 \times (3+d) = -165$$

$$(3-d) \times (3+d) = \frac{-165}{3}$$

$$3^2 - d^2 = -55$$

$$9 - d^2 = -55$$

$$-d^2 = -55 + 9 = -64$$

$$d^2 = 64$$

$$d = \sqrt{64} = \underline{8}$$

$\therefore a-d = 3-8 = -5$

$a+d = 3+8 = 11$

$\therefore$ AP = $\underline{-5, 8, 11}$

QUESTIONS

1. The nth term of a sequence is given by an = 4n +7 .List the first four terms and find the d.
2. Which term of sequence 72 70 68 66 is 40 .
3. The 6^{th} and 17^{th} terms of an AP are 19 and 41 . Find the 40^{th} term
4. How many terms of the sequence 54 51 48 ..... Be taken so that their sum is 513 . Explain the double answer .
5. Find the sum of all integers between 50 and 500 which are divisible by 7
6.

6. The sum of 3 numbers in AP is -3 and their product is 8 . Find the numbers .

7. A man starts repaying a loan as a first instalment of Rs.100 . If he increases the instalment by Rs.5 every month , what amount he will pay in the 30^{th} instalment .

8. Find the sum of odd integers from 1 to 2001 .

9. Find the number of natural numbers between 1 and 100 which are divisible by 3 .Also find the sum of those terms .

1. Find the following if AP , 125 and 155

   (1)Find AM

   (2) Insert 5 terms in between these terms

   (3) Find the sum of these series

1. $a_n = 4n+7$

$a_1 = 4\times1+7 = 11$ $\quad d = 15-11 = 4$

$a_2 = 4\times2+7 = 15$

$a_3 = 4\times3+7 = 19$

$a_4 = 4\times4+7 = 23$

∴ Sequence is 11, 15, 19, 23 and d is 4

2. $a = 72 \quad d = 40-72 = -2$

$40 = 72 + (n-1)-2$

$40 = 72 + -2n + 2$

$40 = 74 - 2n$

$2n = 74-40$

$n = \frac{34}{2} = 17$

3. a_6

$19 = a+(6-1)d$

$a+5d = 19 \longrightarrow ①$

a_{17}

$41 = a+(17-1)d$

$a+16d = 41 \longrightarrow ②$

①−②

$a+5d = 19$

$-\ a+16d = 41$

$-11d = -22$

$d = \frac{-22}{-11} = 2$

Substitute in eq ① $d=2$

$a+5d = 19$

$a+5\times2 = 19$

$a = 19-10 = 9$

a_{40} $\quad a_n = a+(n-1)d$

$= a+(40-1)2$

$= a+39\times2$

$= a+78 = 87$

∴ 40th term is 87

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4. $a = 54 \quad S_n = 513 \quad d = 51-54 = -3$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$513 = \frac{n}{2}[2\times54+(n-1)-3]$

$1026 = n[108+-3n+3]$

$$1026 = n[111 + -3n]$$

$$1026 = 111n - 3n^2$$

$$\frac{3n^2 - 111n + 1026}{3} = 0$$

$$n^2 - 37n + 342 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1$
$b = -37$
$c = 342$

$$= \frac{--37 \pm \sqrt{-37^2 - 4 \times 1 \times 342}}{2 \times 1}$$

$$= \frac{37 \pm \sqrt{1369 - 1368}}{2}$$

$$= \frac{37 \pm \sqrt{1}}{2} = \frac{37 \pm 1}{2}$$

$n = 19$, 18

Number of terms can be 19 or 18

ie, when $n = 19$

$$a_n = 54 + (19-1)-3$$
$$= 54 + 18 \times -3 = 0$$

when $n = 18$

$$a_n = 54 + 17 \times -3 = 3.$$

∴ sum of 19 terms = sum of 18 terms

5. $d = 7$ $a = 56$ $a_n = 497$

$$S_n = \frac{n}{2}[1^{st} \text{ term} + \text{last term}]$$

$$497 = 56 + (n-1)7$$
$$497 = 56 + 7n - 7$$
$$497 = 49 + 7n$$
$$7n = 448$$
$$n = \frac{448}{7} = 64$$

$$S_n = \frac{64}{2}[56 + 497]$$
$$= 32 \times 553 = 17696$$

6. Let the 3 unknown numbers be

$a-d$ a $a+d$

$$a - d + a + a + d = -3$$
$$3a = -3$$
$$a = -1$$

$$(a-d) \times a \times (a+d) = 8$$
$$(-1-d) \times -1 \times (-1+d) = 8$$
$$(-1-d) \times (-1+d) = -8$$
$$-(1^2 - d^2) = -8 \qquad +(1^2 - d^2) = +8$$

$1 - d^2 = -8$

$-d^2 = -8 - 1$

$-d^2 = -9$

$d = 3, -3$

The 3 numbers are

$a - d = -1 - 3 = -4$

$a + d = -1 + 3 = 2$

$\therefore -4, -1, 2$

$a = 100 \quad d = 5 \quad n = 30$

$a_{30} = 100 + 29 \times 5$

$= 100 + 145 = 245$

$\therefore$ AP is 100, 105, 110 ... 245.

$a = 1 \quad a_n = 2001 \quad d = 2$

AP = 1, 3, 5 2001

$2001 = 1 + (n-1)2$

$2001 = 1 + 2n - 2$

$2001 = -1 + 2n$

$2002 = 2n$

$2n = 2002$

$n = 1001$

$S_n = \frac{n}{2}$ [First term + Last term]

$= \frac{1001}{2}[1 + 2001]$

$= 500.5 \times 2002$

$= 1002001$

9. ~~$a = 1 \quad a_n = 100$~~

9. $a = 3 \quad a_n = 99 \quad d = 3$

$a_n = a + (n-1)d$

$99 = 3 + (n-1)3$

$99 = 3 + 3n - 3$

$99 = 3n$

$3n = 99$

$n = 33$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{33}{2}[2 \times 3 + (33-1)3]$

$= 16.5[6 + 32 \times 3]$

$= 16.5 \times 102$

$= 1683$

10. $a = 125 \quad b = 155$

(i) AM $= 125 + 155 / 2 = 140$

(ii) 125 x_1 x_2 x_3 x_4 x_5 155

∴ Number of terms = 7

$155 = 125 + 6d$

$30 = 6d$

$d = 5$

∴ terms are 130, 135, 140, 145, 150

(iii) $S_7 = \frac{7}{2}[2 \times 125 + (7-1)5]$

$= 3.5 [250 + 30]$

$= 980$