

SEQUENCE , SERIES AND PROGRESSION

SUBJECT: BASIC NUMERICAL SKILLS

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DEPT OF COMMERCE

ACADEMIC YEAR :2020-2021

ARITHMETIC PROGRESSION

IMPORTANT POINTS

- **Nth term**

$$a+(n-1)d$$

a : First term

n : Position of the term

d : common difference [$d = a_2 - a_1$]

- **General term**

$$a, a+d, a+2d, a+3d, \dots$$

SUM OF N TERMS OF AN AP

- $S_n = n/2 [2a + (n-1)d]$
when last term is not given

- $S_n = n/2 [\text{first term} + \text{Last term}]$
When last term is given

1. Find the sum of 20 items of the series
3 5 7 9

A. $A=3$ $d=5-3=2$ $n=20$

$$\begin{aligned} S_n &= n/2[2a+(n-1)d] \\ &= 20/2 [2 \times 3 + (20-1)2] \\ &= 10 [6 + 19 \times 2] \\ &= 10 \times 44 \\ &= 440 \end{aligned}$$

2. Find the sum of the series where 1st term is 5 and 15th term is -23.

A. $A=5$ $n=15$ $L_n = -23$

$$\begin{aligned} S_n &= n/2[\text{first term} + \text{last term}] \\ &= 15/2 [5 + (-23)] \\ &= 7.5 \times -18 \\ &= -135 \end{aligned}$$

ARITHMETIC MEAN (AM)

- If a b c are in an AP then b is said to be the AM between a and c

$$AM = (a+b) / 2$$

Eg : Find AM between 5 and 8

$$\begin{aligned} AM &= (5+8)/2 \\ &= 13/2 = 6.5 \end{aligned}$$

ie, 6.5 is the AM between 5 and 8

IF SUM IS GIVEN

- 3 numbers in AP can be assumed as $a-d$,
 a , $a+d$
- 4 numbers in AP can be assumed as $a-3d$,
 $a-d$, $a+d$, $a+3d$

61. Find 3 numbers in AP where Sum is 9 and the product is -165

A. 3 numbers are $a-d$, a , $a+d$

$$S_n = (a-d) + a + (a+d) = 9$$

$$3a = 9$$

$$a = 9/3 = \underline{\underline{3}}$$

Substitute 3 in equation (2).

$$(a-d) \times a \times (a+d) = -165$$

$$(3-d) \times 3 \times (3+d) = -165$$

$$(3-d) \times (3+d) = \frac{-165}{3}$$

$$3^2 - d^2 = -55$$

$$9 - d^2 = -55$$

$$-d^2 = -55 + 9 = -64$$

$$d^2 = 64$$

$$d = \sqrt{64} = \underline{\underline{8}}$$


$$\therefore a-d = 3-8 = -5$$

$$a+d = 3+8 = 11$$

$$\therefore AP = \underline{\underline{-5, 8, 11}}$$

QUESTIONS

1. The n th term of a sequence is given by $a_n = 4n + 7$. List the first four terms and find the d .
2. Which term of sequence $72, 70, 68, 66, \dots$ is 40.
3. The 6th and 17th terms of an AP are 19 and 41. Find the 40th term.
4. How many terms of the sequence $54, 51, 48, \dots$ be taken so that their sum is 513. Explain the double answer.
5. Find the sum of all integers between 50 and 500 which are divisible by 7.
- 6.

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6. The sum of 3 numbers in AP is -3 and their product is 8 . Find the numbers.
 7. A man starts repaying a loan as a first instalment of Rs.100. If he increases the instalment by Rs.5 every month, what amount he will pay in the 30th instalment.
 8. Find the sum of odd integers from 1 to 2001.
 9. Find the number of natural numbers between 1 and 100 which are divisible by 3. Also find the sum of those terms.
 1. Find the following if AP, 125 and 155
 - (1) Find AM
 - (2) Insert 5 terms in between these terms
 - (3) Find the sum of these series
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1. $a_n = 4n + 7$

$$a_1 = 4 \times 1 + 7 = 11$$

$$d = 15 - 11 = 4$$

$$a_2 = 4 \times 2 + 7 = 15$$

$$a_3 = 4 \times 3 + 7 = 19$$

$$a_4 = 4 \times 4 + 7 = 23$$

\therefore sequence is 11, 15, 19, 23 and d is 4

2. $a = 72$ $d = 10 - 72 = -2$

$$40 = 72 + (n-1) \cdot (-2)$$

$$40 = 72 - 2n + 2$$

$$40 = 74 - 2n$$

$$2n = 74 - 40$$

$$n = \frac{34}{2} = \underline{\underline{17}}$$

3. $\frac{a_6}{a_{17}}$

$$19 = a + (6-1)d$$

$$a + 5d = 19 \rightarrow \textcircled{1}$$

$$\frac{a_{17}}$$

$$41 = a + (17-1)d$$

$$a + 16d = 41 \rightarrow \textcircled{2}$$

$$\textcircled{1} - \textcircled{2}$$

$$\begin{array}{r} a + 5d = 19 \\ - \quad a + 16d = 41 \\ \hline -11d = -22 \end{array}$$

$$d = \frac{-22}{-11} = \underline{\underline{2}}$$

Substitute in eq $\textcircled{1}$ $d = 2$

$$a + 5d = 19$$

$$a + 5 \times 2 = 19$$

$$a = 19 - 10 = \underline{\underline{9}}$$

$$\frac{a_{40}}$$

$$a_n = a + (n-1)d$$

$$= 9 + (40-1) \cdot 2$$

$$= 9 + 39 \times 2$$

$$= 9 + 78 = \underline{\underline{87}}$$

\therefore 40th term is 87

4. $a = 54$ $S_n = 513$ $d = 51 - 54 = -3$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$513 = \frac{n}{2} [2 \times 54 + (n-1) \cdot (-3)]$$

$$1026 = n [108 + -3n + 3]$$

$$1026 = n[111 + -3n]$$

$$1026 = 111n - 3n^2$$

$$\frac{3n^2 - 111n + 1026}{3} = 0$$

$$n^2 - 37n + 342 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-37) \pm \sqrt{(-37)^2 - 4 \times 1 \times 342}}{2 \times 1}$$

$$= \frac{37 \pm \sqrt{1369 - 1368}}{2}$$

$$= \frac{37 \pm \sqrt{1}}{2} = \frac{37 \pm 1}{2}$$

$$n = 19, 18$$

Number of terms can be 19 or 18

ie, when $n = 19$

$$a_n = 54 + (19-1) \times -3$$
$$= 54 + 18 \times -3 = \underline{\underline{0}}$$

when $n = 18$

$$a_n = 54 + 17 \times -3 = \underline{\underline{3}}$$

\therefore sum of 19 terms = sum of 18 terms

$$5. \quad d = 7 \quad a = 56 \quad a_n = 497$$

$$S_n = \frac{n}{2} [1^{\text{st}} \text{ term} + \text{last term}]$$

$$497 = 56 + (n-1)7$$

$$497 = 56 + 7n - 7$$

$$497 = 49 + 7n$$

$$7n = 448$$

$$n = \frac{448}{7} = \underline{\underline{64}}$$

$$S_n = \frac{64}{2} [56 + 497]$$

$$= 32 \times 553 = \underline{\underline{17696}}$$

6. let the 3 unknown numbers be

$$a-d \quad a \quad a+d$$

$$a-d + a + a+d = -3$$

$$3a = -3$$

$$a = -1$$

$$(a-d) \times a \times (a+d) = 8$$

$$(-1-d) \times -1 \times (-1+d) = 8$$

$$(-1-d) \times (-1+d) = -8$$

$$-(1^2 - d^2) = -8$$

$$+(1^2 - d^2) = +8$$

$$1-d^2 = -8$$

$$-d^2 = -8-1$$

$$-d^2 = -9$$

$$d = 3, -3$$

The 3 numbers are

$$a-d = -1-3 = -4$$

$$a+d = -1+3 = 2$$

$$\therefore -4, -1, 2$$

$$a=100 \quad d=5 \quad n=30$$

$$a_{30} = 100 + 29 \times 5 \\ = 100 + 145 = \underline{245}$$

\therefore AP is 100, 105, 110, ... 245.

$$a=1 \quad a_n=2001 \quad d=2$$

$$AP = 1, 3, 5, \dots, 2001$$

$$2001 = 1 + (n-1)2$$

$$2001 = 1 + 2n - 2$$

$$2001 = -1 + 2n$$

$$2002 = 2n$$

$$2n = 2002$$

$$n = 1001$$

$$S_n = \frac{n}{2} [\text{First term} + \text{Last term}]$$

$$= \frac{1001}{2} [1 + 2001]$$

$$= 500.5 \times 2002$$

$$= \underline{1002001}$$

9. ~~a=1~~ ~~a=100~~

9. $a=3 \quad a_n=99 \quad d=3$

$$a_n = a + (n-1)d$$

$$99 = 3 + (n-1)3$$

$$99 = 3 + 3n - 3$$

$$99 = 3n$$

$$3n = 99$$

$$n = \underline{33}$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{33}{2} [2 \times 3 + (33-1)3]$$

$$= 16.5 [6 + 32 \times 3]$$

$$= 16.5 \times 102$$

$$= \underline{1683}$$

10. $a=125 \quad b=155$

(i) $AM = \frac{125+155}{2} = \underline{140}$

$$(ii) \quad 125 \quad r_1 \quad r_2 \quad r_3 \quad r_4 \quad r_5 \quad 155$$

\therefore Number of terms n

$$155 = 125 + 6d$$

$$30 = 6d$$

$$\underline{\underline{d = 5}}$$

\therefore terms are 130, 135, 140, 145, 150

$$(iii) \quad \frac{S_n}{125} = \frac{1}{2} [2 \times 125 + (n-1)5]$$
$$= 35 [250 + 30]$$
$$= \underline{\underline{980}}$$