CHAPTER 2

THE REAL NUMBERS

ALGEBRAIC PROPERTIES OF $\mathbb{R}$

- Given any two real numbers a and b , we can form two other real numbers, called their sum a + b and their product a .b and these real numbers satisfy the following axioms
  - commutative property of $\mathbb{R}$

    a + b = b + a for all a , b in $\mathbb{R}$

- Associative property of addition

  $a+(b + c)=(a + b)+c$

- Existence of a zero element

  $a+0=a=0+a$ for all a belongs to $\mathbb{R}$

- Commutative property of multiplication

  $a.b = b.a$ for all a , b belongs to ℝ

- Associative property of multiplication

  $(a.b).c = a.(b.c)$

- Existence of a unit element

  a . 1 = a = 1.a for all a in ℝ

- Existence of reciprocals

  For each a ≠ 0 in ℝ there exists an element 1/a in ℝ such that

  a . (1/a) = (1/a) . a = 1

- Distributive property of multiplication over addition
- $a \,.\, (b + c) = (a \,.\, b) + (a \,.\, c)$ and
- $(b + c).a=(b \,.\, a)+(c \,.\, a)$

  for all a , b and c in $\mathbb{R}$

- Let a , b , c be elements of ℝ

(a) If a>b and b>c, then a>c

(b) If a>b, then a + c > b + c.

(c) If a>b and c>d, then a+ c> b + d

(d) If a>b and c>o, then c a > c b

THEOREM

(a) If z and a are elements in $\mathbb{R}$ With $z + a = a$,

then $z = 0$

(b) If u and b not equal to zero are elements in $\mathbb{R}$

with $u \cdot b = b$, then $u=1$.

(c) a is in $\mathbb{R}$ then $a.0 = 0$

THEOREM

(a) If $a \cdot b = 0$, then either $a = 0$ or $b = 0$.

(b) If $a \neq 0$ and b in $\mathbb{R}$. are such that $a. b =1$, then $b=1/a$.

THANK YOU