Subject: Discrete Mathematics
Topic: Types of graph
Name of the teacher: Lisna Thomas
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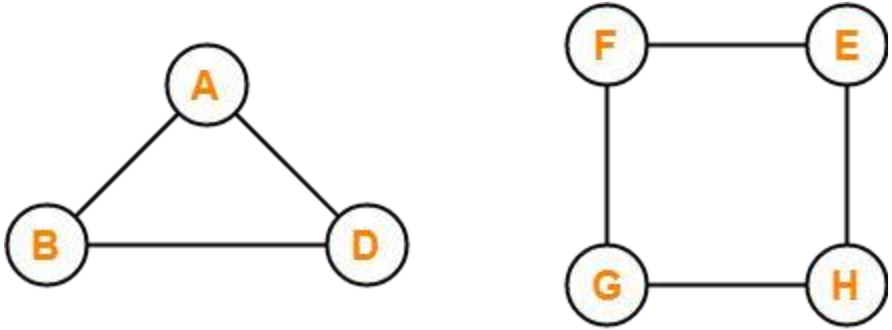
TYPES OF GRAPH

Regular Graph-

- A graph in which degree of all the vertices is same is called as a regular graph.
- If all the vertices in a graph are of degree ‘k’, then it is called as a “**k-regular graph**“.

Examples-



Examples of Regular Graph

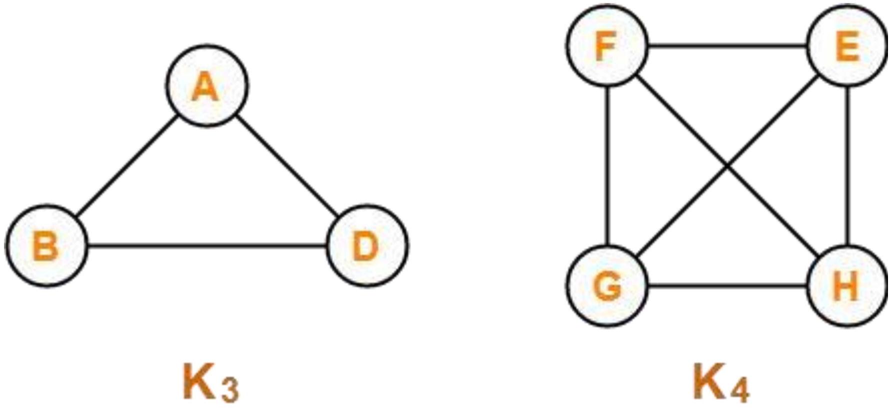
n these graphs,

- All the vertices have degree-2.
- Therefore, they are 2-Regular graphs.

Complete Graph-

- A graph in which exactly one edge is present between every pair of vertices is called as a complete graph.
- A complete graph of ‘n’ vertices contains exactly ${}^{n}C_{2}$ edges.
- A complete graph of ‘n’ vertices is represented as $\mathbf{K_n}$.

Examples-



Examples of Complete Graph

In these graphs,

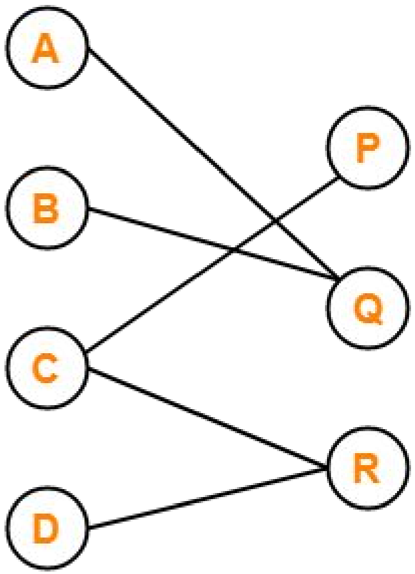
- Each vertex is connected with all the remaining vertices through exactly one edge.
- Therefore, they are complete graphs.

Bipartite Graph-

A bipartite graph is a graph where-

- Vertices can be divided into two sets X and Y.
- The vertices of set X only join with the vertices of set Y.
- None of the vertices belonging to the same set join each other.

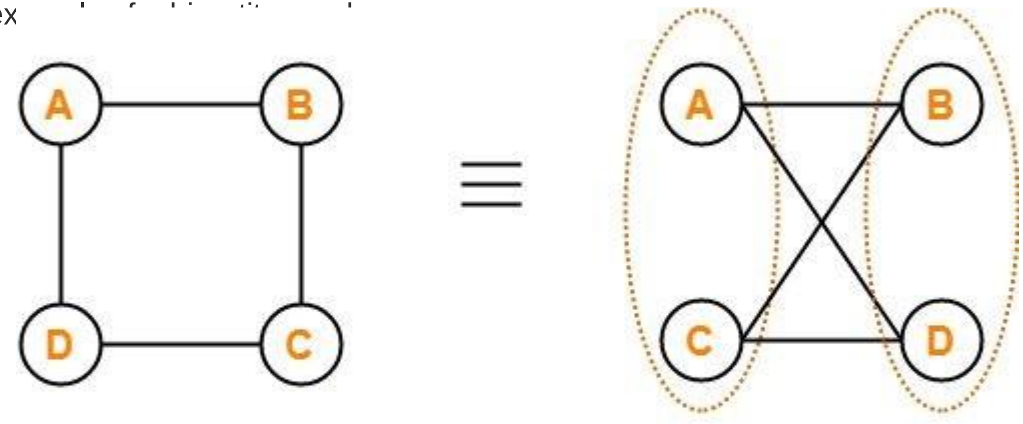
Example-



Example of Bipartite Graph

Bipartite Graph Example-

The following graph is an ex



Example of Bipartite Graph

Here,

- The vertices of the graph can be decomposed into two sets.
- The two sets are X = {A, C} and Y = {B, D}.
- The vertices of set X join only with the vertices of set Y and vice-versa.
- The vertices within the same set do not join.
- Therefore, it is a bipartite graph.

Complete Bipartite Graph-

A complete bipartite graph may be defined as follows-

A bipartite graph where every vertex of set X is joined to every vertex of set Y

is called as complete bipartite graph.

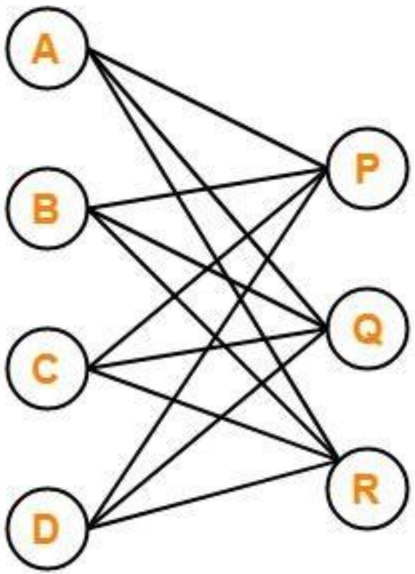
OR

Complete bipartite graph is a bipartite graph which is complete.

OR

Complete bipartite graph is a graph which is bipartite as well as complete.

Complete Bipartite Graph Example-



Example of Complete Bipartite Graph

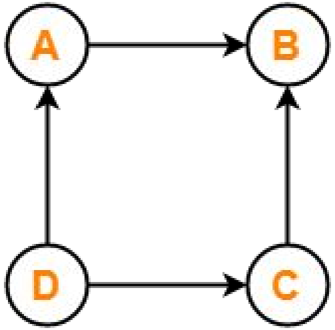
Here,

- This graph is a bipartite graph as well as a complete graph.
- Therefore, it is a complete bipartite graph.
- This graph is called as $\mathbf{K_{4,3}}$.

Directed Graph-

- A graph in which all the edges are directed is called as a directed graph.
- In other words, all the edges of a directed graph contain some direction.
- Directed graphs are also called as **digraphs**.

Example-



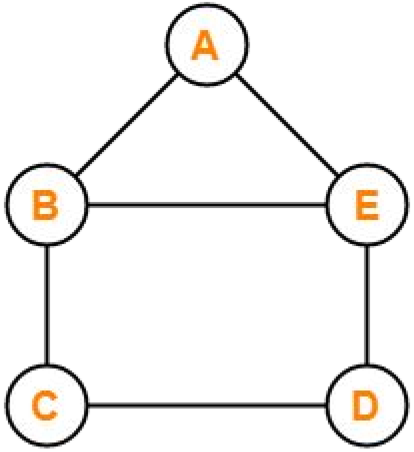
Example of Directed Graph

Here,

- This graph consists of four vertices and four directed edges.
- Since all the edges are directed, therefore it is a directed graph.

Planar Graph-

- A planar graph is a graph that we can draw in a plane such that no two edges of it cross each other.



Example of Planar Graph

Here,

- This graph can be drawn in a plane without crossing any edges.
- Therefore, it is a planar graph.

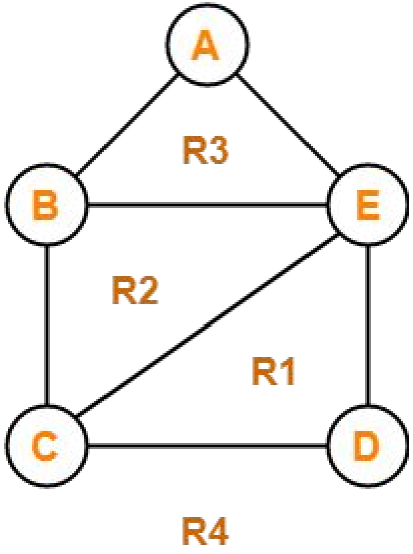
Regions of Plane-

The planar representation of the graph splits the plane into connected areas called as **Regions of the plane**.

Each region has some degree associated with it given as-

- Degree of Interior region = Number of edges enclosing that region
- Degree of Exterior region = Number of edges exposed to that region

Example-



Regions of Plane

Here, this planar graph splits the plane into 4 regions- R1, R2, R3 and R4 where-

- Degree (R1) = 3
- Degree (R2) = 3
- Degree (R3) = 3
- Degree (R4) = 5

Planar Graph Properties-

Property-01:

In any planar graph, Sum of degrees of all the vertices = 2 x Total number of edges in the graph

Property-02:

In any planar graph, Sum of degrees of all the regions = 2 x Total number of edges in the graph

Euler Graph-

- Euler Graph is a connected graph in which all the vertices are even degree.

An Euler graph may be defined as-

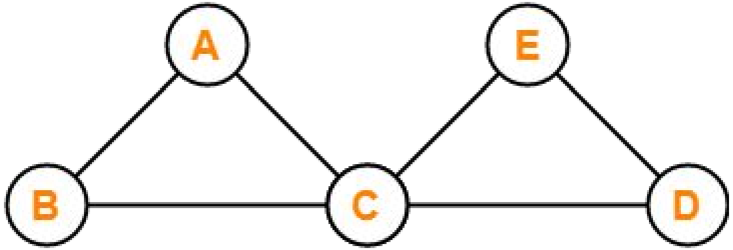
Any connected graph is called as an Euler Graph if and only if all its vertices are of even degree.

OR

An Euler Graph is a connected graph that contains an Euler Circuit.

Euler Graph-

Example-



Example of Euler Graph

Here,

- This graph is a connected graph.
- The degree of all the vertices is even.
- Therefore, it is an Euler graph.

Euler Path-

Euler path is also known as **Euler Trail** or **Euler Walk**.

- If there exists a **Trail** in the connected graph that contains all the edges of the graph, then that trail is called as an Euler trail.

OR

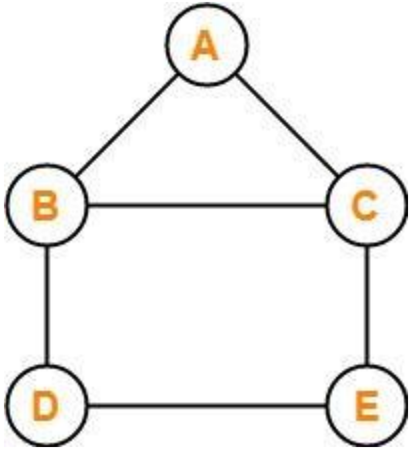
- If there exists a walk in the connected graph that visits every edge of the graph exactly once with or without repeating the vertices, then such a walk is called as an Euler walk.
-

Walk in Graph Theory-

- A walk is defined as a finite length alternating sequence of vertices and edges.
- The total number of edges covered in a walk is called as **Length of the Walk**.

Walk in Graph Theory Example-

Consider the following graph-



In this graph, few examples of walk are-

- a , b , c , e , d (Length = 4)
- d , b , a , c , e , d , e , c (Length = 7)
- e , c , b , a , c , e , d (Length = 6)

Open Walk in Graph Theory-

In graph theory, a walk is called as an **Open walk** if-

- Length of the walk is greater than zero
- And the vertices at which the walk starts and ends are different.

Closed Walk in Graph Theory-

In graph theory, a walk is called as a **Closed walk** if-

- Length of the walk is greater than zero
- And the vertices at which the walk starts and ends are same.

Path in Graph Theory-

In graph theory, a path is defined as an open walk in which-

- Neither vertices (except possibly the starting and ending vertices) are allowed to repeat.
- Nor edges are allowed to repeat.

Cycle in Graph Theory-

In graph theory, a cycle is defined as a closed walk in which-

- Neither vertices (except possibly the starting and ending vertices) are allowed to repeat.
- Nor edges are allowed to repeat.

OR

In graph theory, a closed path is called as a cycle.

Trail in Graph Theory-

In graph theory, a trail is defined as an open walk in which-

- Vertices may repeat.
- But edges are not allowed to repeat.

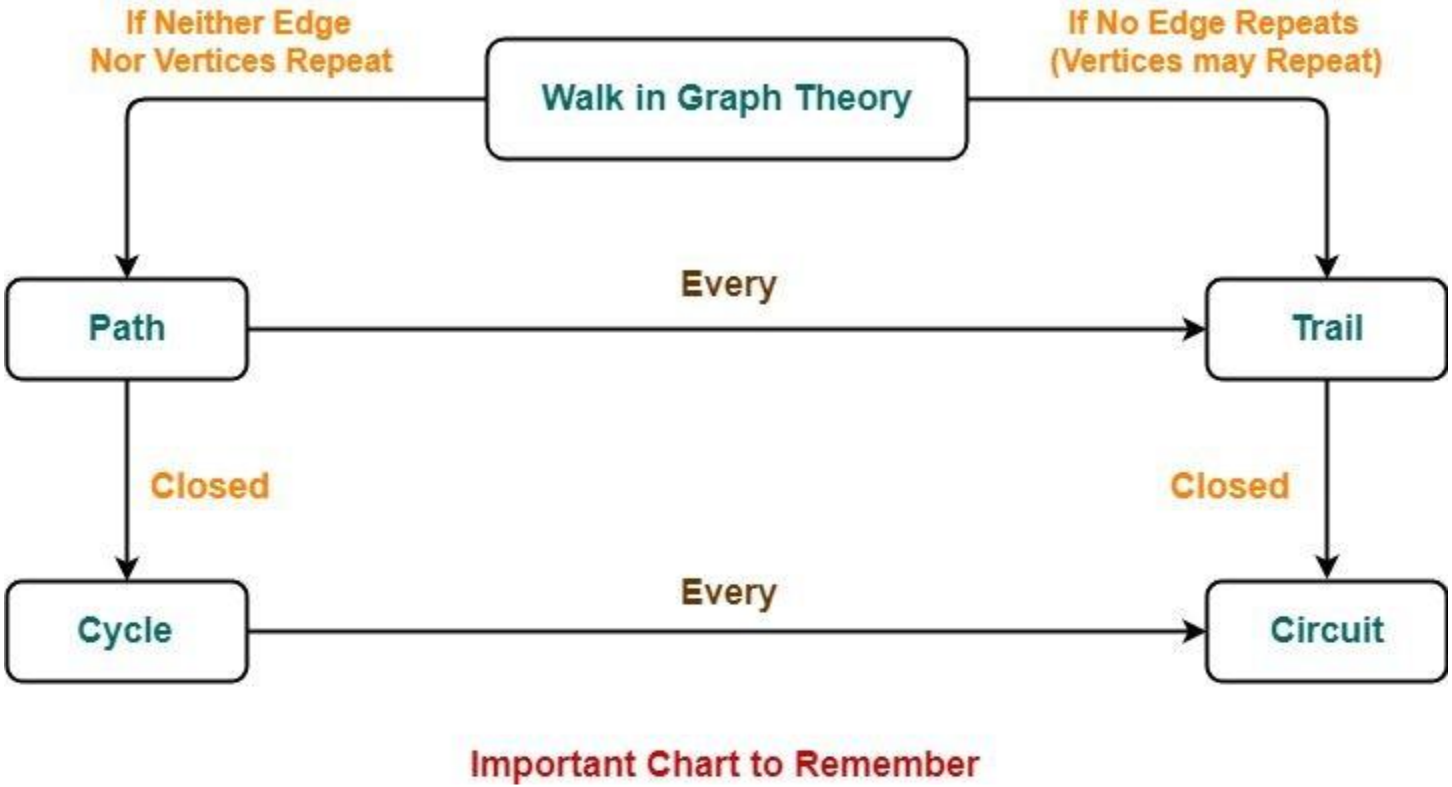
Circuit in Graph Theory-

In graph theory, a circuit is defined as a closed walk in which-

- Vertices may repeat.
- But edges are not allowed to repeat.

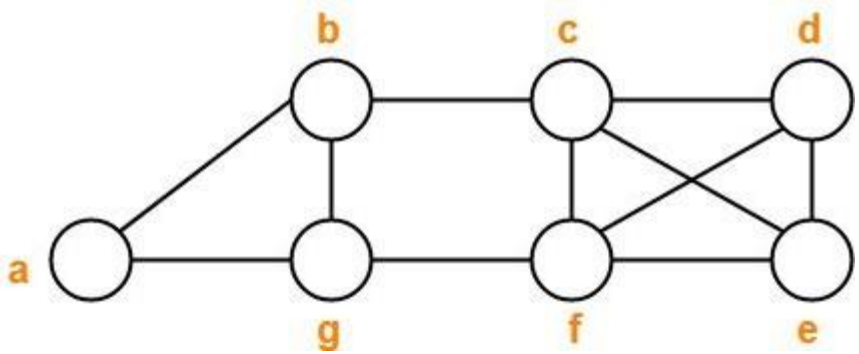
OR

In graph theory, a closed trail is called as a circuit.


If Neither Edge
Nor Vertices Repeat
Walk in Graph Theory
If No Edge Repeats
(Vertices may Repeat)
Path
Every
Trail
Closed
Closed
Cycle
Every
Circuit
Important Chart to Remember

Problem-01:

Consider the following graph-



Decide which of the following sequences of vertices determine walks.

For those that are walks, decide whether it is a circuit, a path, a cycle or a trail.

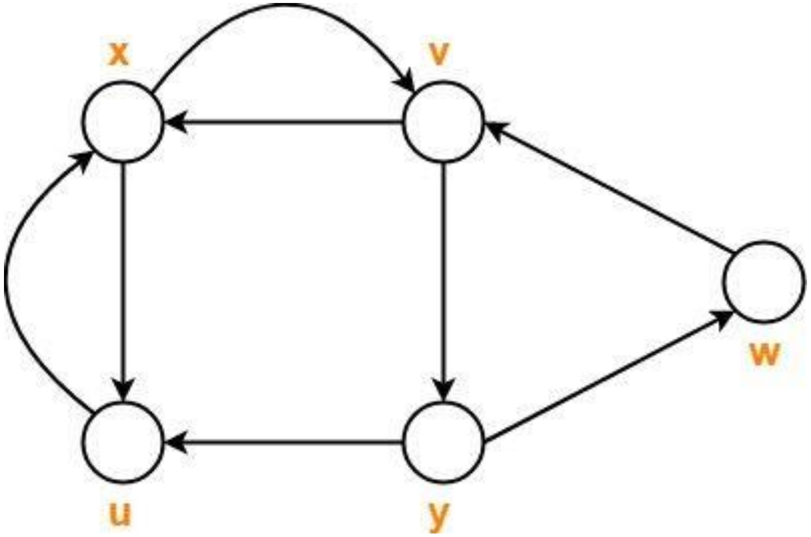
1. a , b , g , f , c , b
2. b , g , f , c , b , g , a
3. c , e , f , c
4. c , e , f , c , e
5. a , b , f , a
6. f , d , e , c , b

Solution-

1. Trail
2. Walk
3. Cycle
4. Walk
5. Not a walk
6. Path

Problem-02:

Consider the following graph-



Consider the following sequences of vertices and answer the questions that follow-

1. x , v , y , w , v
2. x , u , x , u , x
3. x , u , v , y , x
4. x , v , y , w , v , u , x

Decide which of the following sequences of vertices determine walks.

For those that are walks, decide whether it is a circuit, a path, a cycle or a trail.

Euler Circuit-

Euler circuit is also known as **Euler Cycle** or **Euler Tour**.

- If there exists a **Circuit** in the connected graph that contains all the edges of the graph, then that circuit is called as an Euler circuit.

OR

- If there exists a walk in the connected graph that starts and ends at the same vertex and visits every edge of the graph exactly once with or without repeating the vertices, then such a walk is called as an Euler circuit.

OR

- An Euler trail that starts and ends at the same vertex is called as an Euler circuit.

OR

- A closed Euler trail is called as an Euler circuit.