SEQUENCE, SERIES AND PROGRESSION :-HARMONIC PROGRESSION

SUBJECT: BASIC NUMERICAL SKILLS

STEFY M M
DEPT OF COMMERCE

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Harmonic progression

- A harmonic progression [HP] is defined as a sequence of real numbers which is determined by taking the reciprocals of the arithmetic progression that does not contain 0. in the HP, any term in the sequence is considered as the harmonic means of its two neighbours.

  e.g. The sequence a, b, c, d,..... Is considered as an AP the HP can be calculated as 1/a, 1/b,1/c, 1/d,......
- Harmonic mean: HM is calculated as the reciprocal of the AM of the reciprocals. The formula to calculated the harmonic mean is given by:

  HM=n/{(1/a)+(1/b)+(1/c)+(1/d)+.........}

  where,

  a, b, c, d, =values &

  n= the no. of value present.

Harmonic progression formula

"The nth term of the HP= 1/{a+(n-1)d}"

Where

a= first term of AP

d= the common difference

n= the number of terms in AP.

The Hp formula is also written as:

"Nth term of HP= 1/(nth term of the corresponding AP)"

Sum of harmonic progression

- If the HP is (1/a), (1/a + d), (1/a+2d),……the formula to find the sum of n term is
- Sn =1/d*In{[2n+(2n-1)d]/[2a-d]}
- where,
  - a= the 1st term of AP
  - d= the common different of AP
  - In= the natural log ration.

Relation Between AP, GP and HP

For any two numbers, if A.M, G.M, H.M are the Arithmetic, Geometric, and Harmonic Mean respectively, then the relationship between these three is given by:

- $G.M^2 = A.M \times H.M$, where A.M, G.M, H.M are in G.P
- $A.M \geq G.M \geq H.M$

Harmonic Progression Examples

Here, solved problems on the harmonic progression are given.

Example 1:

Determine the 4th and 8th term of the harmonic progression 6, 4, 3,…

Solution:

Given:

H.P = 6, 4, 3

Now, let us take the arithmetic progression from the given H.P

A.P = ⅙, ¼, ⅓, ….

Here, $T_2 - T_1 = T_3 - T_2 = 1/12 = d$

So, in order to find the 4th term of an A. P, use the formula,

The nth term of an A.P = a+(n-1)d

Here, a = ⅙, d= 1/12

Now, we have to find the 4th term.

So, take n=4

Now put the values in the formula.

4th term of an A.P = (⅙) +(4-1)(1/12)

= (⅙)+(3/12)

= (⅙)+ (¼)

= 5/12

Similarly,

8th term of an A.P = (⅙) +(8-1)(1/12)

= (⅙)+(7/12)

= 9/12

Since H.P is the reciprocal of an A.P, we can write the values as:

4th term of an H.P = 1/4th term of an A.P = 12/5

8th term of an H.P = 1/8th term of an A.P = 12/9 = 4/3

Example 2:

Compute the 16th term of HP if the 6th and 11th term of HP are 10 and 18 respectively.

Solution:

The H.P is written in terms of A.P are given below:

nth term of A.P = a+5d = 1/10 —- (!)

11th term of A.P = a+10d = 1/18 ……(2)

By solving these two equations, we get

a =13/90, and d = -2/ 225

To find 16th term, we can write the expression in the form,

a+15d = (13/90) – (2/15) = 1/90

Thus, the 16th term of an H.P = 1/16th term of an A.P = 90

Therefore, the 16th term of the H.P is 90.

Extra questions

1. If the sum of reciprocals of first 11 terms of an HP series is 110, find the 6^{th} term of HP.
2. Find the 4th and 8th term of the series 6, 4, 3, …
3. The 2nd term of an HP is 40/9 and the 5th term is 20/3. Find the maximum possible number of terms in H.P.
4. **Find the infinite sum of series $1/(3^2-4) + 1/(4^2-4) + 1/(5^2-4)$ …….**

Answers:

1. Now from the above HP formulae, it is clear the reciprocals of first 11 terms will make an AP. The sum of first 11 terms of an AP = [2a + (11 – 1) d] 11/2 = 110 $\Rightarrow 2a + 10d = 20 \Rightarrow a + 5d = 10$
   Now there are 2 variables, but $a + 5d = T_6$ in an AP series. And reciprocal of 6th term of AP series will give the 6th term of corresponding HP series. So, the 6th term of HP series is 1/10.
2. Consider 1/6, /14, 1/3, ...
   Here $T_2 – T_1 = T_3 – T_2 = 1/12$
   Therefore 1/6, 1/4, 1/3 is in A.P.
   4th term of this Arithmetic Progression = 1/6 + 3 × 1/12 = 1/6 + 1/4 = 5/12,
   Eighth term = 1/6 + 7 × 1/12 = 9/12.
   Hence the 8th term of the H.P. = 12/9 = 4/3 and the 4th term = 12/5.

Answer: Option B

Explanation : If a, a + d, a + 2d, a + 3d, ... are in A.P. then $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \frac{1}{a+3d}$, ... are in H.P.

Now $\frac{1}{a+d} = \frac{40}{9} \Rightarrow a+d = \frac{9}{40}$ and $\frac{1}{a+4d} = \frac{20}{3} \Rightarrow a+4d = \frac{3}{20}$

Solving these two equations we get $a = \frac{1}{4}$ and $d = \frac{-1}{40}$

Now $a + 10d = \frac{1}{4} + \frac{-10}{40} = 0$

Hence the maximum terms that this H.P. can take is 10.

4. In the given series,

$S= 1/(3^2-4) + 1/(4^2-4) + (1/5^2-4) + \ldots..$

$S= 1/(3^2 - 2^2) + 1/(4^2-2^2) + 1/(5^2 - 2^2) + \ldots..$

We know, that $a^2-b^2 = (a-b)(a+b)$

$S= 1/[(3-2)(3+2)] + 1/[(4-2)(4+2)] + 1/[(5-2)(5+2)] + \ldots..$

$S = \frac{1}{4}(1-1/5) + \frac{1}{4}(1/2 - 1/6) + \frac{1}{4}(1/3 - 1/7)$

$S = \frac{1}{4} * 1.833$

$S = 0.45$

Questions:

1) Arti walked first one-fourth of the distance at a speed of 2 km/hr. The next one-fourth of the distance was covered by running at the speed of 3km/hr. The last one-fourth of the distance was covered by cycling at the speed of 6 km/hr. Find the average speed for the whole journey covered by Arti?
2) How many terms of GP 3,32,33..are needed to give the sum 120?
3) Given a GP with a=729 and 7^{th} term 64, determine S7.
4) x^3, x^5, x^7,……..n terms (if $x \neq \pm 1$).

Answer

1. According to the question, the distance covered is same in all the three cases.

   Therefore the average speed = HM of 2, 3, and 6

   Average speed = 4 / (1/2 + 1/3 + 1/6)

   Average speed = 4 / 1

   Average speed = 4 km/hr.
2. 4
3. 2059
4. $X^3 (1-x^{2n})/(1+x^2)$

THANK YOU