CANTILEVER

LALY A.S.
ASST. PROFESSOR
DEPT. OF PHYSICS
LITTLE FLOWER COLLEGE,
GURUVAYUR

A cantilever is a thin uniform bar fixed horizontally at one end and loaded at other end.

Two cases:

1.Bending of the beam due to the weight of the beam is not to be considered.

2.Bending due to the weight of the beam also be considered.

Case1:When the weight of the beam is ineffective

Let AB represents the neutral axis of a cantilever of length L,fixed at the end A and loaded at end B with a weight W.The end B is deflected into the position B’ and the neutral axis takes the position AB’.

Consider a section PQ of the beam at a distance x from the fixed end A.

Deflecting couple due to the load W at this section=W(L-x)

Restoring couple=YI/R

At equilibrium W(L-x)=YI/R

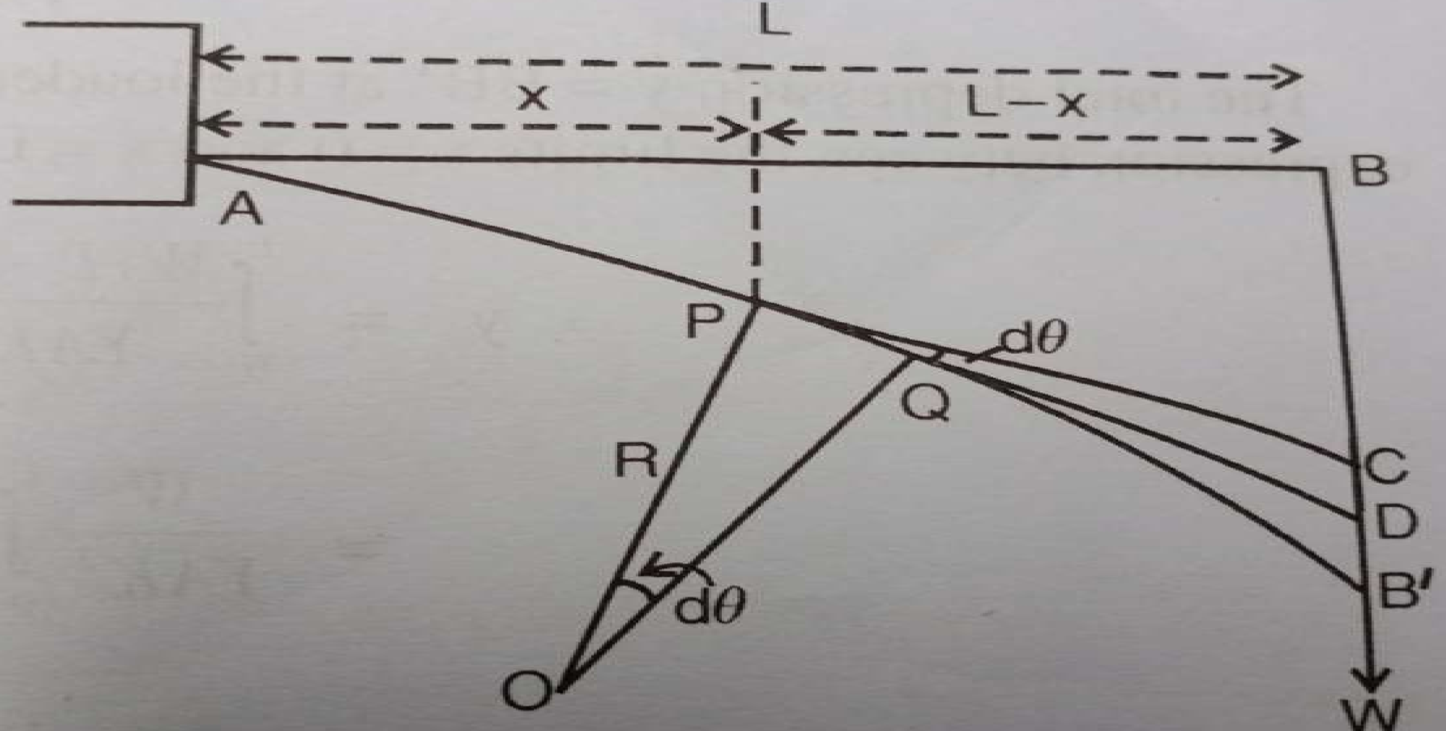

Fig.8

$W(L-x)=YAK^2/R$ -------(1)

From the figure ,dx=R dθ or R=dx/dθ

$$W(L-x)=YAK^2\frac{d\theta}{dx}$$

$$d\theta=\frac{W(L-x)dx}{YAK^2}\quad ------(2)$$

The tangents from P and Q meets at C and Don the vertical line BB'.The angle subtended by the tangents is also equal to dθ.

The depression CD can be represented by dy.

$$dy=(L-x)d\theta$$

Substituting from equation(2)

$$dy = \frac{(L-x)W(L-x)dx}{YAK^2} = \frac{W(L-x)^2 dx}{YAK^2}$$

The Total depression y=BB' at the loaded end B is obtained by integrating the above expression between the limits x=0 and x=L

$$\therefore y = \int_0^L \frac{W(L-x)^2 dx}{YAK^2} \quad = \frac{W}{YAK^2}\int_0^L (L^2 - 2Lx + x^2)dx$$

$$y = \frac{W}{YAK^2}\left[L^2 x - \frac{2Lx^2}{2} + \frac{x^3}{3}\right]_0^L$$

$$y = \frac{W}{YAK^2}\left[L^3 - L^3 + \frac{L^3}{3}\right]$$

$$y = \frac{WL^3}{3YAK^2} = \frac{WL^3}{3YI}$$

When the weight of the beam is not considering ,

The free end of the cantilever is depressed by $\frac{WL^3}{3YI}$

Case2:Cantilever loaded uniformly

In this case, no weight is added to the end B.Let the weight per unit length is W_1.The weight of the portion (L-x)of the beam is W_1(L-x).

The weight W_1(L-x)acts at a distance (L-x)/2 from the section PQ.Bending moment due to weight of the beam is

$$W_1(L-x)\frac{(L-x)}{2}=\frac{W_1}{2}(L-x)^2$$

At equilibrium bending moment will be equal to the restoring moment

$$\frac{YI}{R}=\frac{W_1}{2}(L-x)^2$$

$$\frac{YI}{R} = \frac{W_1}{2}(L-x)^2$$

$$We\ know\ R = \frac{dx}{d\theta}$$

$$\frac{YI\ d\theta}{dx} = \frac{W_1}{2}(L-x)^2$$

$$d\theta = \frac{W_1(L-x)^2\,dx}{2YI}$$

$$Also\ dy = (L-x)d\theta$$

$$\therefore \frac{dy}{(L-x)} = d\theta$$

$$Substituting\ d\theta, we\ have$$

$$dy = \frac{W_1(L-x)^3\,dx}{2YI}$$

Total depression will be obtained by integrating the above expression within the limit x=0 and x=L

$$y = \frac{W_1}{2YI}\int_0^L (L-x)^3\,dx = \frac{W_1 L^4}{8YI}$$

Since $w_1L=W_2$,the weight of the beam

$$y = \frac{W_2 L^3}{8YI}$$

Special cases:

1. <u>RECTANGULAR CROSS SECTION</u>

Moment of inertia $I = \frac{bd^3}{12}$ where b=breadth and d=thickness of the beam

Depression at the loaded end,

$$y = \frac{WL^3}{3YI} = \frac{MgL^3}{3YI} = \frac{4MgL^3}{Ybd^3}$$

2.CIRCULAR CROSS SECTION

Moment of inertia $I = \frac{\pi r^4}{4}$ where r is the radius

Depression at the loaded end,

$$y = \frac{MgL^3}{3YI} = \frac{4MgL^3}{3Y\pi r^4}$$

ASSIGNMENT

Obtain the expression for y considering the weight of the beam and loaded weight at one end .

THANK YOU……