“STUDY ON THE STRUCTURE PROPERTIES OF M-FUZZY GROUPS AND FUZZY G-MODULES”

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BY

MS. ALPHY JOSE

(PRINCIPAL INVESTIGATOR)

ASST. PROFESSOR, DEPT. OF MATHEMATICS

LITTLE FLOWER COLLEGE, GURUVAYOOR, KERALA-680103

SUMMARY OF THE PROJECT

TOPIC: "STUDY ON THE STRUCTURE PROPERTIES OF M-FUZZY GROUPS AND FUZZY G-MODULES"

In 1965 Zadeh introduced the notion of a fuzzy subset μ of a non empty set X as a function from X to unit interval $I = [0,1]$. The notion of fuzzy groups was introduced by Rosenfeld in 1971. Fuzzification of classical concepts such as groups, rings, modules etc. opened up a new insight in the field of mathematical sciences.

Definition: Fuzzy Group

A fuzzy subset μ on a group G is called a fuzzy subgroup of G if

(1) $\mu(xy) \geq \mu(x) \wedge \mu(y)$

(2) $\mu(x) = \mu(x^{-1})$

Proposition:

Let μ be a fuzzy subgroup of a group G. Then $\mu_* = \{x \in G: \mu(x) = \mu(e)\}$ where e is the identity in G, will be a subgroup of G.

Note: If μ is a fuzzy set then the $\alpha - cut$ of μ is defined as the crisp set

$\mu_\alpha = \{x \in G : \mu(x) \geq \alpha\}$

Proposition:

If μ is a fuzzy subgroup of a group G, then each $\alpha - cut$of μ is a subgroup of G.

Definition: Support of μ

The support of a fuzzy set μ is defined as $\mu^* = \{x \in G: \mu(x) > 0)\}$

Theorem:

The support of a fuzzy subgroup μ on is also a fuzzy subgroup

Definition: M-group

Let G be a group and M be any set. G is called an M-group if for any $g \in G$ and $m \in M$ there exists a product $mg \in G$such that $m(gh) = (mg)(mh)$ for all $g, h \in G$ & $m \in M$.

Example 1: Let $G = (R^n, +)$ and M be a subset of natural number.

Define for $m \in M, x = (x_1, x_2, \ldots \ldots \ldots \ldots x_n) \in R^n$, $mx = (mx_1, mx_2, \ldots \ldots \ldots \ldots mx_n) \in R^n$.

Then G is an M-group.

Definition: M- fuzzy group

Let G be a M-group and μ be a fuzzy group on G. μ is called an M-fuzzy group if $\mu(mg) \geq \mu(g)$ for all $g \in G$ & $m \in M$.

Example: Let $G = (R^n, +)$ be the M-group defined in Example 1. Define a fuzzy group μ on G by $\mu(x) = \begin{cases} 0, & if\ at\ least\ one\ x_j \neq 0 \\ 1, & otherwise \end{cases}$

Then $\mu(mx) = \mu(x), x \in R^n\ and\ m \in M$. Hence μ is an M-fuzzy group on G.

Proposition:

If μ is a M-fuzzy group on G, then each $\alpha - cut\ \mu_\alpha$ of μ is a M-group.

Proof:

Let $m \in M$, $g \in \mu_\alpha$

$\mu(mg) \geq \mu(g) \geq \alpha \Rightarrow mg \in \mu_\alpha$

Also $m(gh) = (mg)(mh)\ \forall m \in M\ \&\ g, h \in G$

Thus $\alpha - cut\ \mu_\alpha$ is a M-group which is a subgroup of G.

Proposition:

The support μ^* of a M-fuzzy group μ is also a M-group.

Theorem

If μ is a M-fuzzy group on G then μ^n is also an M- fuzzy group on G where $\mu^n(g) = \left(\mu(g)\right)^n\ \ \forall g \in G$

Proposition:

Let G be a M-group and μ, ϑ be two M-fuzzy subgroups on G, then $\mu \cap \vartheta$ is also a M-fuzzy subgroup of G where $(\mu \cap \vartheta)(x) = \mu(x) \wedge \vartheta(x)$.

Definition: Normal fuzzy subgroup

A fuzzy subgroup μ of a group G is called Normal fuzzy subgroup if $\mu(x^{-1}yx) \geq \mu(y) \forall x, y \in G$.

Definition: *M*-normal fuzzy subgroup

Let G be a M-group. A fuzzy subgroup μ of G is said to be a M-normal fuzzy subgroup of G if μ is a M-fuzzy subgroup and also a normal fuzzy subgroup of G.

Proposition:

Let G be a M-group and μ, ϑ be two M-normal fuzzy subgroups on G, then $\mu \cap \vartheta$ is also a M- normal fuzzy subgroup of G.

Definition: M-homomorphism

If G & G^1 are M-groups and f is a homomorphism from Gonto G^1 such that $f(mg) = m\,f(g)$ for all $m \in M$ & $g \in G$ then $f$is called an M-homomorphism.

Example: Let $G = (R^n, +)$ be the M-group defined in Example 1. Also $G^1 = (R, +)$ is an M-group with the operation $m(x + y) = mx + my$ for $m \in M\ and\ x, y \in R$.Define $f: R^n \to R$ by $f(x) = \sum_{j=1}^{n} x_j$. Then $f(mx) = m.f(x)\ \forall x \in G\ and\ m \in M$ so that f is an M-homomorphism.

Let G & G^1be M-groups and let f be an M-homomorphism from Gonto $G^1$and μbe an M-fuzzy group on G. Then $f(\mu)$ is an M- fuzzy group on G^1 where

$f(\mu)(y) = \vee \{\mu(x): x \in f^{-1}(y), y \in R(f)\}$. Also if ϑ is a M-fuzzy group on G^1 then $f^{-1}(\vartheta)$ is a M-fuzzy group on G where $f^{-1}(\vartheta)(x) = \vartheta(f(x))$.

If μ is a M-fuzzy group on G then μ^n is also an M- fuzzy group on G where $\mu^n = \{(g, (\mu(g))^n): g \in G\}$.

Definition: G-Module

Let G be a finite group and M be a vector space over the field Kwhich is a subfield of $\mathbb{C}$. Then M is a G $-$module if for all $g \in G$ & $m \in M$ there exists $gm \in M$such that

1) $em = m\ \forall\, m \in M$ where e is the identity in G
2) $(gh)m = g(hm)\ \forall g \in G$ & $m \in M$
3) $g(k_1m_1 + k_2m_2) = k_1(gm_1 + gm_2)$

Example: Let $G = \{1, -1\}$ & $M = R^4$ over R. Define $g.(x_1, x_2, x_3, x_4) = (gx_1, gx_2, gx_3, gx_4) \in M$ for $g \in G\ and\ (x_1, x_2, x_3, x_4) \in M$. Then M is a G-module.

Definition: Fuzzy G-Module

Let G be a finite group and M be G $-$module over the field Kwhich is a subfield of $\mathbb{C}$.Then a fuzzy G $-$module on M is a fuzzy set μ of M such that

1) $\mu(ax + by) \geq \mu(x) \wedge \mu(y)\ \forall a, b \in K$ & $x, y \in M$
2) $\mu(gm) \geq \mu(m)\ \forall g \in G$ & $m \in M$

Example: Let M be the G-module defined in previous example.

Define $\mu(x) = \begin{cases} 1,\ if\ x_i = 0\ \forall\ i \\ 1/2,\ if\ at\ least\ one\ x_i \neq 0 \end{cases}$ where $(x_1, x_2, x_3, x_4) \in M$. Then μ is a fuzzy G-module on M.

Definition: Fuzzy submodule

Let μ be a fuzzy set of a G −module M. Then μ is called a fuzzy submodule of M if

1) $\mu(0) = 1$ where 1 is the additive identity in M
2) $\mu(gm) \geq \mu(m) \;\; \forall g \in G \;\&\; m \in M$
3) $\mu(x + y) \geq \mu(x) \wedge \mu(y) \;\; \forall\, x, y \in M$

Let μ be a fuzzy submodule of a G −module M and if we define $x \equiv y(mod\, \mu)$ $\Leftrightarrow \mu(x - y) = \mu(0) = 1$ denoted by $x\mu^*y$ then μ^* is an equivalence relation. We can easily show that it is reflexive, symmetric and transitive.

Every submodule of a G −module M induces an equivalence relation. Also we can show that if $x\mu^*y$ then $\mu(x) = \mu(y)$.

Let $\mu^*[x]$ be the equivalence class containing $x \in M$, where M is a G − module. Then $M/_{\mu} = \{\mu^*[x]: x \in M\}$, the set of all equivalence classes. Defining two operations $\oplus$ *and* $*$ in $M/_{\mu}$ as $\mu^*[x] \oplus \mu^*[y] = \mu^*[x + y]$ and $r * \mu^*[x] = \mu^*[rx]$ where $x, y \in M \;\&\; r \in K$ we can make $M/_{\mu}$ a vector space over K.

Also $(M/_{\mu}, \oplus, *)$ is a G −module if M is a G −module by defining the product of $\mu^*[x] \in M/_{\mu}$ and $g \in G$ as $g.\mu^*[x] = \mu^*[gx]$.

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