Summary of the project

Topic**: Study of homomorphism and factor group of fuzzy soft group**

Most of the existing mathematical tools for formal modeling, reasoning and computing are crisp, deterministic and precise in character. But in real life situation, the problems in economics, engineering, social science etc. do not always involve crisp data. P K Maji initiated the concept of fuzzy soft set theory . A fuzzy soft set is denoted by the pair (f, A) where f is a mapping from A to set of all fuzzy sets in X. In my present project I have introduced a basic version of fuzzy soft groups theory which extends the notion of a group to the algebraic structures of fuzzy soft sets. If X is a group and (f,A) is a fuzzy soft set over x is called a fuzzy soft group if and only if for each aϵ A and for every x ,yϵ X

$f_a(xy) \geq \min(f_a(x), f_a(y))$ (1)

$f_a(x^{-1}) \geq f_a(x)$ (2)

Example

Let X = {1,i,-1,-i} and A= {a_1,a_2,a_3}. X is a group under multiplication.

Define $f{:}A \rightarrow I^X$ as

(f,A) = {f(a1)={1/8, i/.4, -1/.6, -i/.4},

f(a2)={1/.3, i/.1, -1/,2,-i/.1},

f(a3)={1/.9,i/.3,-1/.6, -1/.3}}

Then the pair (f, A) satisfies the conditions (1) and (2). Then (f, A) is a fuzzy soft group over X. Also I have studied the properties and structural characteristics of fuzzy soft groups and normal fuzzy soft groups. Further more definitions of left and right cosets, middle coset of a fuzzy soft group are defined, and some of its basic properties are studied.

Let (f, A) be the fuzzy soft group over X.

Left coset of (f, A) determined by $x \in X$, $(f, A)_{L(x)}$ is defined as for each $a \in A, (f_a)_{L(x)}(y) = f_a(x^{-1}y), \forall y \in X$.

Right coset of (f, A) determined by $x \in X$, $(f, A)_{R(x)}$ is defined as for each $a \in A, (f_a)_{R(x)}(y) = f_a(yx^{-1}), \forall y \in X$.

Consider the above example

Right coset of (f, A) determined by -1 is given by

$(f,A)_{R(-1)}$ = { $f(a_1)_{R(-1)}$ = { 1/.6, i/.4, -1/.8, -i/.4},

$f(a_2)_{R(-1)}$ = { 1/.2, i/.1, -1/.3, -i/.1},

$f(a_3)_{R(-1)}$ = { 1/.6, i/.3, -1/.9, -i/.3}}

Let (f,A) be the fuzzy soft group over X , then middle coset of (f,A) determined by $x, y \in X$, $(f,A)_{x,y}$ is defined as for each

$a \in A, (f_a)_{x,y}(z) = f_a(x^{-1}zy^{-1}), \forall z \in X$.

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Middle coset of (f,A) determined by 1, i in above example is given by

$(f,A)_{1,i} = \{ \; f(a_1)_{1,i} = \{ 1/.4, i/.8, -1/.4, -i/.6\},$

$\qquad f(a_2)_{1,i} = \{ 1/.1, i/.3, -1/.1, -i/.2\},$

$\qquad f(a_3)_{1,i} = \{ 1/.3, i/.9, -1/.3, -i/.6\}\}$

Finally I have defined the concept of factor group of a fuzzy soft group.

Let (f,A) and (g,B) be fuzzy soft groups over X and Y respectively and

$\varphi: X \to Y$ and $\psi: A \to B$ be two functions, where A and B are parameter sets for the crisp sets X and Y respectively. Then the fuzzy soft function

$(\Phi,\Psi) : X \to Y$ is defined as $(\Phi(f), \Psi(A))$,

$$\text{where } \Phi(f)_k(y) = \begin{cases} \bigvee_{\Phi(x)=y} \bigvee_{\Psi(a)=k} f_a(x), \text{ if } x \in \Phi^{-1}(y); \\ 0, \text{ otherwise} \end{cases}$$

$\forall k \in \Psi(A), \forall y \in Y$

The pre-image of (g, B) under the fuzzy soft function (Φ,Ψ) is defined as

$$(\Phi,\Psi)^{-1}(g,B) = (\Phi^{-1}(g), \Psi^{-1}(B))$$

$$\text{Where } \Phi^{-1}(g)_a(x) = g_{\Psi(a)}(\Phi(x)), \quad \forall a \in \Psi^{-1}(B), \quad \forall x \in X$$

Further, more fuzzy soft homomorphism is defined and some of its basic properties are studied.