RELATIVITY - PROBLEMS

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1. In Michelson Morely experiment, the distance from the partially silvered glass plate to each of the mirrors was 11m. If the wavelength of the light used was 6000Å and the expected fringe shift was 0.4 , find the velocity of earth relative to ether.

Given l=11m $\quad \lambda = 6000\text{Å} = 6000 \times 10^{-10}\ m \quad \Delta n$=0.4 $\quad v$=?

$$\frac{2lv^2}{c^2} = \Delta n\lambda \Rightarrow v^2 = \frac{\Delta n\lambda c^2}{2l} = \frac{0.4\times6000\times10^{-10}\times(3\times10^8)^2}{2\times11}$$

$$= \frac{0.4\times6000\times10^{-10}\times9\times10^{16}}{2\times11} = \frac{0.4\times6\times9\times10^9}{2\times11} = 0.9818 \times 10^9 = 9.818 \times 10^8$$

$$v = \sqrt{v^2} = \sqrt{9.818 \times 10^8} = 3.133 \times 10^4 m/s$$

2.A rocket is 40 metre long on the ground. When it is in flight its length is 38 metre to an observer on the ground. Find the speed of the rocket.

- $l_0 = 40m$ $\quad\quad$ $l = 38m$ $\quad\quad$ $v = ?$

$$l = l_0\sqrt{1-\frac{v^2}{c^2}} \Rightarrow \frac{l}{l_0} = \sqrt{1-\frac{v^2}{c^2}} \Rightarrow \left(\frac{l}{l_0}\right)^2 = 1-\frac{v^2}{c^2}$$

$$\Rightarrow \frac{v^2}{c^2} = 1-\left(\frac{l}{l_0}\right)^2 \Rightarrow \frac{v}{c} = \sqrt{1-\left(\frac{l}{l_0}\right)^2}$$

$$\Rightarrow \mathrm{v} = c\sqrt{1-\left(\frac{l}{l_0}\right)^2} = c\sqrt{1-\left(\frac{38}{40}\right)^2} = 0.312c$$

3.A particle with mean proper life time of $\mathbf{2 \times 10^{-6}}$sec moves through the laboratory with a speed of 0.99c. Calculate its life time as measured by an observer in laboratory.

$\Delta t_0 = 2 \times 10^{-6}\text{sec}$ v=0.99c

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{2 \times 10^{-6}}{\sqrt{1-\frac{(0.99c)^2}{c^2}}}$$

$$= \frac{2 \times 10^{-6}}{\sqrt{1-0.99^2}} = 14.17 \times 10^{-6} sec$$

4.A particle with mean proper life of 1μs moves through the laboratory at $\mathbf{2.7 \times 10^8} \boldsymbol{m}/\mathrm{s}$. What will be the distance travelled by it before disintegration?

$\Delta t_0 = \mathbf{1\mu s} = 1 \times 10^{-6}\text{sec}$

v= $\mathbf{2.7 \times 10^8} \boldsymbol{m/s}$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{10^{-6}}{\sqrt{1-\frac{(2.7\times10^8)^2}{(3\times10^8)^2}}} = 2.29 \times 10^{-6} sec$$

$$\therefore \boldsymbol{distance\ travelled\ x = v\ \Delta t} = \mathbf{2.7 \times 10^8} \times 2.3 \times 10^{-6}$$ = **621m.**

5. μ-meson has mean life of 2μs. What is the mean life when it moving with a speed 0.98c. How far does it travel in this time?

$\Delta t_0 = 2\mu\text{s} = 2 \times 10^{-6}\text{sec}$ v= $\mathbf{0.98c}$

$\Delta t = \dfrac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \dfrac{2 \times 10^{-6}}{\sqrt{1 - \frac{(\mathbf{0.98c})^2}{c^2}}} = 10.05 \times 10^{-6} sec$

$\therefore \boldsymbol{distance\ travelled\ x = v}\, \Delta t = \mathbf{0.98 \times 3 \times 10^8} \times 10.05 \times 10^{-6}$= **2954.81m.**

6.Anu is 25 years old and her father Anand is 60 years old. Anand goes to space in a spaceship. On returning from space, Anand finds himself 70 years old whereas his daughter has become 65 years old. Find the velocity of the spaceship.

$\Delta t_0 = 70 - 60 = 10$years $\qquad \Delta t$=65-25=40years

V=?

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \sqrt{1 - \frac{v^2}{c^2}} = \frac{\Delta t_0}{\Delta t} \Rightarrow 1 - \frac{v^2}{c^2} = \left(\frac{\Delta t_0}{\Delta t}\right)^2$$

$$\Rightarrow 1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2 = \frac{v^2}{c^2} \Rightarrow \sqrt{1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2} = \frac{v}{c}$$

$$\Rightarrow v = c\sqrt{1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2} = 3 \times 10^8 \sqrt{1 - \left(\frac{10}{40}\right)^2} = 2.905 \times 10^8 \text{ m/s.}$$

7.Nikhil is 20 years old and his father Vinod is 50 years old. Vinod travels in a spaceship and comes back to earth. On returning from the space ship, Vinod finds himself only 60 years old whereas his son has become 70 years old on earth. Calculate the velocity of the ship.

$\Delta t_0 = 60 - 50 = 10$years Δt=70-20=50years

V=?

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \sqrt{1-\frac{v^2}{c^2}} = \frac{\Delta t_0}{\Delta t} \Rightarrow 1-\frac{v^2}{c^2} = \left(\frac{\Delta t_0}{\Delta t}\right)^2$$

$$\Rightarrow 1-\left(\frac{\Delta t_0}{\Delta t}\right)^2 = \frac{v^2}{c^2} \Rightarrow \sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2} = \frac{v}{c}$$

$$\Rightarrow v = c\sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2} = 3\times 10^8\sqrt{1-\left(\frac{10}{50}\right)^2} = 2.939\times 10^8 \text{ m/s.}$$

8.A rocket is 100 meter long on earth. When it is in flight, its length is 98m to an observer in the space laboratory. Compute the speed of rocket.

- $l_0 = 100m$ l=98m v=?

$$l= l_0 \sqrt{1-\frac{v^2}{c^2}} \Rightarrow \frac{l}{l_0} = \sqrt{1-\frac{v^2}{c^2}} \Rightarrow \left(\frac{l}{l_0}\right)^2 = 1-\frac{v^2}{c^2}$$

$$\Rightarrow \frac{v^2}{c^2} = 1-\left(\frac{l}{l_0}\right)^2 \Rightarrow \frac{v}{c} = \sqrt{1-\left(\frac{l}{l_0}\right)^2}$$

$$\Rightarrow \mathrm{v} = c\sqrt{1-\left(\frac{l}{l_0}\right)^2} = c\sqrt{1-\left(\frac{98}{100}\right)^2} = 0.199c = 0.596 \times 10^8 \text{ m/s.}$$

9.A space craft of length 125m and diameter 10m is moving with a speed of 0.98c. What is its length and diameter w.r.t. an observer on earth?

- $l_0 = 125m$ $d_0 = 10m$ $v = o.98c$

$$l = l_0\sqrt{1-\frac{v^2}{c^2}} \Rightarrow l = 125\sqrt{1-\frac{(0.98c)^2}{c^2}}$$

$\Rightarrow l = 125\sqrt{1-0.98^2}$ =24.875m.

Diameter remains same since perpendicular directions are unaffected.

10.A 200 m long train has to pass through 150m long tunnel. If the train moves towards the tunnel with a speed of 0.8c, find the length of the train and the tunnel as seen by a man (a) at the train (b) at the tunnel

l_0(train)=200m l_0(tunnel)=150m v=0.8c

For a man at the train

Length of the train= its proper length=200m

Length of the tunnel= $l_0(\text{tunnel})\sqrt{1-\frac{v^2}{c^2}}=150\sqrt{1-0.8^2}$=90m

For a man at the tunnel

Length of the tunnel = its proper length=150m

Length of the train= $l_0(\text{train})\sqrt{1-\frac{v^2}{c^2}}=200\sqrt{1-0.8^2}$=120m

11. Compute the speed of a rocket whose clock runs one second slower per hour relative to a clock on the earth.

No. of seconds in 1 hour = 60x60=3600sec

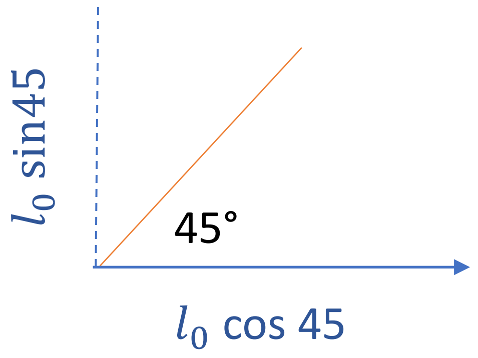
Δt=3600 sec Δt_0 =3599sec V=?

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \sqrt{1-\frac{v^2}{c^2}} = \frac{\Delta t_0}{\Delta t} \Rightarrow 1-\frac{v^2}{c^2} = \left(\frac{\Delta t_0}{\Delta t}\right)^2$$

$$\Rightarrow 1-\left(\frac{\Delta t_0}{\Delta t}\right)^2 = \frac{v^2}{c^2} \Rightarrow \sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2} = \frac{v}{c}$$

$$\Rightarrow v = c\sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2} = 3\times 10^8\sqrt{1-\left(\frac{3599}{3600}\right)^2} = 7.07\times 10^6 \text{ m/s.}$$

12. A uniform rod of certain length is moving horizontally with a velocity of 0.8c making an angle 45° with the direction of motion. Calculate the percentage contraction in length of the rod.



The length of the rod can be resolved into $l_0$cos 45 and $l_0$sin 45.

Since the rod is moving along x-axis, contraction takes place in the cosine component.

$$l_x = l_0 \cos 45 \sqrt{1 - \frac{v^2}{c^2}} = \frac{l_0}{\sqrt{2}} \sqrt{1 - 0.8^2} = \frac{0.6 l_0}{\sqrt{2}}$$

Since no contraction takes place in the y direction, $l_y = l_0 \sin 45 = \frac{l_0}{\sqrt{2}}$

Effective length of the rod $l = \sqrt{l_x^2 + l_y^2} = \sqrt{\left(\frac{0.6 l_0}{\sqrt{2}}\right)^2 + \left(\frac{l_0}{\sqrt{2}}\right)^2} = \sqrt{0.36 \frac{l_0^2}{2} + \frac{l_0^2}{2}} = l_0 \sqrt{0.68} = 0.825\, l_0$

Percentage of contraction in length = $\frac{l_0 - l}{l_0} \times 100 = \frac{l_0 - 0.825 l_0}{l_0} \times 100 = 17.54\%$

13. Show that $x^2 + y^2 + z^2 - c^2t^2$ is Lorentz invariant

$$x'^2 + y'^2 + z'^2 - c^2t'^2 \quad = \left(\frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}}\right)^2 + y^2 + z^2 - c^2\left(\frac{t-\frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}\right)^2$$

$$= \frac{x^2-2xvt+v^2t^2}{1-\frac{v^2}{c^2}} + y^2 + z^2 - c^2\left(\frac{t^2-\frac{2xvt}{c^2}+\frac{v^2x^2}{c^4}}{1-\frac{v^2}{c^2}}\right)$$

$$= \frac{x^2-2xvt+v^2t^2}{1-\frac{v^2}{c^2}} + y^2 + z^2 - \frac{c^2t^2-2xvt+\frac{v^2x^2}{c^2}}{1-\frac{v^2}{c^2}}$$

$$= \frac{x^2-2xvt+v^2t^2-c^2t^2+2xvt-\frac{v^2x^2}{c^2}}{1-\frac{v^2}{c^2}} + y^2 + z^2 \quad = \frac{x^2\left(1-\frac{v^2}{c^2}\right)-c^2t^2\left(1-\frac{v^2}{c^2}\right)}{1-\frac{v^2}{c^2}} + y^2 + z^2$$

$$= x^2 + y^2 + z^2 - c^2t^2$$

14. At what speed v, will the Galilean and Lorentz expressions for x differ by 10%?

$x_G = x - vt$ and $x_L = \frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}}$

Since $x_L > x_G$, we have to find out the speed for which $\frac{x_L - x_G}{x_G}$ = 10% = $\frac{10}{100} = 0.1$

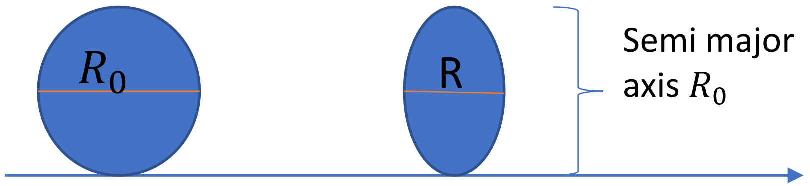
$$\frac{\left(\frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}}\right) - (x-vt)}{(x-vt)} = 0.1 \Rightarrow \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 = 0.1 \Rightarrow 1.1 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Squaring on both sides, $1.21 = \frac{1}{1-\frac{v^2}{c^2}} \Rightarrow 1 - \frac{v^2}{c^2} = \frac{1}{1.21} = 0.8264$

$$\Rightarrow \frac{v^2}{c^2} = 1 - 0.8264 = 0.1736 \Rightarrow \frac{v}{c} = \sqrt{0.1736} = 0.4167 \Rightarrow v = 0.4167c$$

15. A circular ring in x-y plane moves parallel to the x-axis. What should be its velocity so that its area appears to be half the stationary area

Area of the ring = πR_0^2, R_0 *be the radius of the ring*.



As it moves along x- direction, its radius along x-axis suffers contraction. As a result, the ring assumes the shape of an ellipse with semi major radius R_0 and semi minor radius R.

Area of the ellipse = $\pi\ R_0$ R.

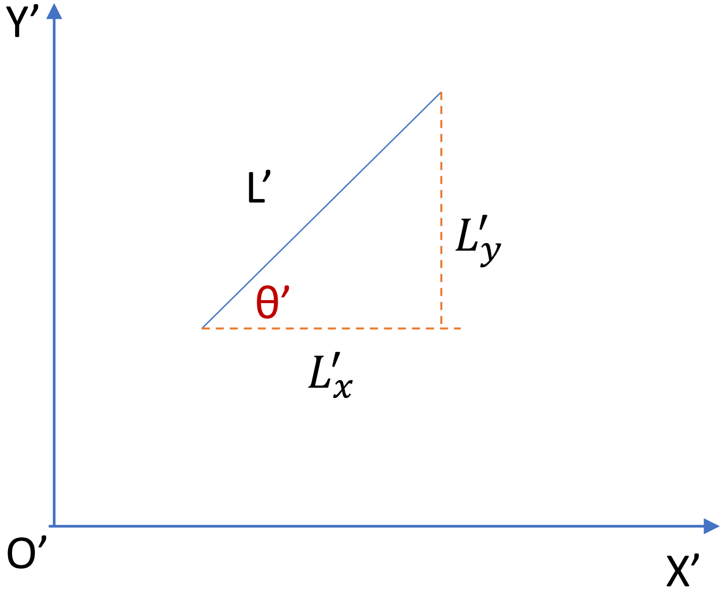
As this area appears to be half of the ring, we have $\pi\ R_0\ R = \frac{\pi R_0^2}{2} \Rightarrow R = \frac{R_0}{2}$

$$l = l_0\sqrt{1-\frac{v^2}{c^2}} \Rightarrow \frac{R_0}{2} = R_0\sqrt{1-\frac{v^2}{c^2}} \Rightarrow \frac{1}{2} = \sqrt{1-\frac{v^2}{c^2}}$$

$$\Rightarrow \frac{1}{4} = 1-\frac{v^2}{c^2} \Rightarrow \frac{v^2}{c^2} = \frac{3}{4} \Rightarrow \frac{v}{c} = \frac{\sqrt{3}}{4}$$

$\Rightarrow v = \frac{\sqrt{3}}{2}$c = 0.866c = 2.598 x 10^8m/s.

16. Frame S’ moves with velocity v relative to a frame S. A rod in frame S’ makes an angle θ’ with respect to the forward direction of motion. Show that the angle θ as measured in S related to θ’ by $cot\theta = \left(1 - \frac{v^2}{c^2}\right)^{1/2} cot\theta'$



Let L’ be the length of the rod in the S’ frame.

Using Pythagorus theorem, L’=$\left({L'_x}^2 + {L'_y}^2\right)^{1/2}$.

L'_x=L’ cos θ’ and L'_y=L’ sin θ’

$\frac{L'_x}{L'_y} = \cot\theta'$

Let L be the length of the rod in the S frame.

Using Pythagorus theorem, L=$\left(L_x^2 + L_y^2\right)^{1/2}$.

L_x=L cos θ and L_y =L sin θ

$\frac{L_x}{L_y} = \cot\theta$

Using $l= l_0\sqrt{1-\frac{v^2}{c^2}}$, $L_x = L'_x\sqrt{1-\frac{v^2}{c^2}}$ and $L_y = L'_y$.

$\frac{L_x}{L_y} = \frac{L'_x}{L'_y}\sqrt{1-\frac{v^2}{c^2}}$ $\Rightarrow$ $\cot\theta = \cot\theta'\sqrt{1-\frac{v^2}{c^2}}$

Problem 17: A photon is moving with the velocity of light c in an inertial frame S', which also moves with a uniform velocity v. Show that the velocity of the photon remains the same.

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}} = \frac{c + v}{1 + \frac{cv}{c^2}}$$

$$= \frac{c + v}{1 + \frac{v}{c}} = \frac{c + v}{\frac{c + v}{c}} = c$$

18.Calculate the speed at which the mass of an electron becomes 4 times its rest mass.

m= $\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ => 4 $m_0 = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$

=> 4= $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ => 16= $\frac{1}{1-\frac{v^2}{c^2}}$

$\Rightarrow$1- $\frac{v^2}{c^2}$= $\frac{1}{16}$ => $\frac{15}{16} = \frac{v^2}{c^2}$

$\Rightarrow$=> $\mathrm{v} = \frac{\sqrt{15}}{4}\mathrm{c}$ = 2.905$\times$ $10^8 m/s$

19.The average life time of muons at rest is 2.4×10^{-6}s. Its average life time measured in a laboratory is $6 \times 10^{-6} s$. Find (a) the speed of the muons in the laboratory (b) its effective mass at the speed (c) KE . Given rest mass = m_0

$\Delta t = 6 \times 10^{-6} s$ $\quad\quad$ $\Delta t_0 = 2.4 \times 10^{-6}$s

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow v = c\sqrt{1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2} = c\sqrt{1 - \left(\frac{2.4}{6}\right)^2} = 0.9165c$$

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{m_0}{\sqrt{1 - \frac{0.9165^2 c^2}{c^2}}} = \frac{m_0}{\sqrt{1 - 0.9165^2}} = 2.499\, m_0$$

$$KE = mc^2 - m_0 c^2 = 2.499\, m_0 c^2 - m_0 c^2 = 1.499\, m_0 c^2$$

$$= 1.499 \times 9 \times 10^{16}\, m_0 = 13.49 \times 10^{16}\, m_0 \text{ J}$$

20.Calculate the mass of an electron accelerated to a kinetic energy of 2MeV. $m_0 = 9.1 \times 10^{-31} kg$

$$KE = mc^2 - m_0\, c^2$$

$$mc^2 = KE + m_0\, c^2$$

$$m = \frac{KE}{c^2} + m_0$$

$$= \frac{2\times10^6\times1.6\times10^{-19}}{(3\times10^8)^2} + 9.1\times 10^{-31}$$

$$= 44.65 \times 10^{-31}\text{kg}$$

21. A particle is moving with a speed of 0.4c. Find the ratio of the rest mass and the mass in motion.

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \frac{m_0}{m} = \sqrt{1-\frac{v^2}{c^2}} = \sqrt{1-\frac{0.4^2c^2}{c^2}} = 0.916$$

22.A stationary bomb explodes into two fragments of rest mass 1kg each moving apart with a speed of 0.6c. Find the rest mass of the bomb.

The total energy of the stationary bomb= Total energy of the fragments.

$$m_0c^2 = \frac{m_{01}c^2}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{m_{02}c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

Given m_{01}= m_{02}=1kg, v=0.6c

$$m_0c^2 = \frac{1\times c^2}{\sqrt{1-\frac{0.6^2c^2}{c^2}}} + \frac{1\times c^2}{\sqrt{1-\frac{0.6^2c^2}{c^2}}} = \frac{2c^2}{\sqrt{1-0.6^2}} = \frac{2c^2}{\sqrt{0.64}}$$

m_0=2.5kg

23. Calculate the mass of a particle whose kinetic energy is half of its total energy. Find its speed.

Total energy = mc^2

Kinetic energy = mc^2- m_0c^2

Given, KE=TE/2

ie, mc^2- m_0c^2=$\frac{mc^2}{2}$ => $\frac{mc^2}{2}$ = m_0c^2 => $\frac{m}{2}$= m_0

$\Rightarrow m$=2 m_0

m= $\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ => 2 m_0 = $\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ => 2= $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ => $\sqrt{1-\frac{v^2}{c^2}}$ = $\frac{1}{2}$

=> 1$-\frac{v^2}{c^2}$= $\frac{1}{4}$ => $\frac{v^2}{c^2}$=1- $\frac{1}{4}$ = $\frac{3}{4}$ => v= $\frac{\sqrt{3}}{2}$c=0.866c

24. Find the increase in mass of 1kg of a metal of specific heat 0.15cal/g/K when heated through $800°C$.

Energy required to increase the temperature of 1gm of metal through 1K=0.15 cal

Energy required to increase the temperature of 1kg(1000gm) of metal through 1K=0.15x1000 cal

Energy required to increase the temperature of 1kg(1000gm) of metal through 800K=0.15x1000x800 cal

energy exachanged for increase of $800°C$ is H=mst

=1000$\times$ $0.15 \times 800\ cal$=1.2$\times$ $10^5 cal$

= 1.2$\times$ 10^5 $\times$4.2 J = 5.04 $\times$ 10^5J

But E=mc^2 => increase in mass =$\frac{E}{c^2}$ = $\frac{5.04 \times 10^5}{(3\times10^8)^2}$=5.6 $\times$ $10^{-12} kg$

25. A proton of rest mass 1.67$\times$ $10^{-27}kg$ moves with a velocity $\frac{c}{\sqrt{2}}$. $Find\ its\ mass$, momentum, total energy and kinetic energy.

$$m= \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_0}{\sqrt{1-\frac{\left(\frac{c}{\sqrt{2}}\right)^2}{c^2}}} = \frac{m_0}{\sqrt{1-\frac{\left(\frac{1}{\sqrt{2}}\right)^2}{1}}} = \frac{m_0}{\sqrt{1-\frac{1}{2}}} = \frac{m_0}{\sqrt{\frac{1}{2}}} =\sqrt{2}\, m_0$$

$= \sqrt{2} \times 1.67\times 10^{-27}$ = 2.36×10^{-27} kg

Momentum p= mv = $2.36 \times 10^{-27} \times \frac{c}{\sqrt{2}}$ = $5.006 \times 10^{-19} kgm/s$

Total energy = mc^2=$2.36 \times 10^{-27} c^2$ = $21.24 \times 10^{-11} J$

Kinetic energy=Total energy-rest energy= mc^2- $m_0\ c^2$= $\sqrt{2}\, m_0\ c^2$- $m_0\ c^2$ = $0.414\ m_0\ c^2$=$6.222 \times 10^{-11} J$

26. A square of side of length 'a' is moving with a speed c/2 parallel to one of its sides. What is its area in motion?

Using the equation $l = l_0\sqrt{1-\frac{v^2}{c^2}}$,

we can write a'$= a\sqrt{1-\frac{v^2}{c^2}}$ = a $\sqrt{1-\frac{c^2}{4c^2}}$ = a $\sqrt{1-\frac{1}{4}}$ = a $\sqrt{\frac{3}{4}}$ = 0.866a

Contraction takes place only in one side. The newlength after contraction is a'.

The length of the other side remains the same and is equal to a.

Therefore area in motion= axa'=0.866 a^2

27. How much younger an astronaut will appear to the earth observer if he returns after one year having moved with a velocity 0.8c

Δt=1yr

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \Delta t_0 = \Delta t\sqrt{1-\frac{v^2}{c^2}} = 1\sqrt{1-\frac{(0.8c)^2}{c^2}} = \sqrt{1-0.8^2}$$

=$\sqrt{0.36}$ =0.6 years

The astronaut is younger by Δt- Δt_0=1-0.6=0.4years

28. A star is 10 light years away from earth. How long would it take for a spaceship travelling with a velocity 3×10^6 to reach the star with respect to an observer (a) on earth (b) on the spaceship

(a) Time to reach the star w.r.t. earth's frame= $\Delta t = \frac{l}{v}$

$$= \frac{10\times365\times24\times60\times60\times3\times10^8 m}{3\times10^6 m/s} \text{ (in second)} = \frac{\frac{10\times365\times24\times60\times60\times3\times10^8}{3\times10^6}}{365\times24\times60\times60} \text{ years}$$

$$= \frac{10\times3\times10^8}{3\times10^6} \text{ years} = 1000 \text{ years}$$

(b) According to time dilation $\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}$

Time to reach the star w.r.t. spaceship, $\Delta t_0 = \Delta t \sqrt{1-\frac{v^2}{c^2}}$

$$= 1000 \sqrt{1-\frac{(3\times10^6)^2}{(3\times10^8)^2}} = 999.9 \text{ years}$$

29. Consider two identical twins of age 25 years. One remains on earth and the other travels within a spaceship with a velocity $\frac{\sqrt{3}}{2}c$. After 25 years elapsed on earth, traveller returns. Then what are their ages?

$time\ elapsed\ on\ earth\ \Delta t$=25yrs

Time elapsed in spaceship $\Delta t_0 = \Delta t\sqrt{1-\frac{v^2}{c^2}}$ = 25 $\sqrt{1-\frac{3}{4}}$ =12.5years.

Age of the traveller = 25+12.5 = 37.5 years

Age of the one who stayed on earth = 25+ 25=50 years

30. An electron falls through a potential difference of 10^4volts. Calculate the percentage increase in mass . $m_0 = 9 \times 10^{-31} kg$.

When an electron of charge e falls through a potential difference of V volts,

the kinetic energy acquired K = eV = $1.6 \times 10^{-19} \times 10^4$= 1.6×10^{-15}J

From the equation K=(m- m_0)c^2, m- m_0=$\frac{K}{c^2}$= $\frac{1.6\times10^{-15}}{9\times10^{16}}$=1.8 $\times$ 10^{-32}kg.

∴Percentage increase in mass = $\frac{\text{m}- m_0}{m_0}$ ×100 = $\frac{1.8 \times10^{-32}}{9\times10^{-31}}$ ×100=2%

31. Calculate the energy equivalent of 1kg of coal.

M=1kg

E=m c^2=1$\times$ $(3 \times 10^8)^2$=9 $\times$ 10^{16}J

32. Prove that when v/c<<1, the relativistic kinetic energy becomes the classical one.

Relativistic kinetic energy K= m$c^2 - m_0c^2 = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}c^2 - m_0c^2$

= $m_0c^2\left(1-\frac{v^2}{c^2}\right)^{-1/2} - m_0c^2$

when v/c<<1, $\left(1-\frac{v^2}{c^2}\right)^{-1/2}$ =1+ $\frac{v^2}{2c^2}$

$\therefore$ K= $m_0c^2\left(1+\frac{v^2}{2c^2}\right) - m_0c^2$ = $\frac{m_0\ v^2}{2}$.

This is the classical kinetic energy.