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BY
MS. DARY JOHN
(Principal Investigator)

ASSISTANT PROFESSOR IN MATHEMATICS
DEPARTMENT OF MATHEMATICS
LITTLE FLOWER COLLEGE
GURUVAYOOR,THRISSUR , KERALA-680103

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A STUDY ON FUZZY SOFT RINGS

1. INTRODUCTION

In many complicated problems arising in the fields of engineering, social science, economics, medical science etc involving uncertainties, classical methods are found to be inadequate in recent times. Soft Set Theory, initiated by Molodtsov is free of the difficulties present in these theories. In 2011, Neog and Sut put forward a new notion of complement of a soft set and accordingly some important results have been studied in their work. In recent times, researches have contributed a lot towards fuzzification of Soft Set Theory. Maji et alintroduced the concept of Fuzzy Soft Set and some properties regarding fuzzy soft union, intersection, complement of a fuzzy soft set, De Morgan Law etc. In this paper, I have studied some properties of fuzzy soft rings.

2. PRELIMINARIES

Definition2.1[Fuzzy Set

Let U be the initial universe set. Then a fuzzy set is a function F from U to [0,1]. i.e.,

$$F : U \rightarrow [0,1]$$

If F(u)= u_i , then u_i is known as the membership grade of u with respect to the fuzzy function F.

Definition2.2Soft Set

Let U be the initial universe set and E be the set of parameters. . Let $A \subseteq E$ and P(U) denotes the power set of U. A pair (F,A) is called a soft set over U ,where F is a mapping given by $F : A \rightarrow P(U)$

Definition2.3Fuzzy Soft Set

Let U be the initial universe set and E be the set of parameters. Let $A \subseteq E$. A pair (F,A) is called a fuzzy soft set over U ,where F is a mapping given by $F : A \to [0,1]^U$, where $[0,1]^U$ denotes the collection of all fuzzy subsets of U.

Definition 2.4Fuzzy Soft Group

Let G be a group and (F,A) be a fuzzy soft set over *G*. Then (F,A) is said to be a fuzzy soft group over G if and only if for each $a \in A$ and $x,y \in G$

(1) $F_a(x.y) \geq \min\{ F_a(x) , F_a(y)\}$

(2) $F_a(x^{-1}) \geq F_a(x)$, where F_a is the fuzzy subset of G corresponding to the parameter $a \in A$.

Definition 2.5 Fuzzy Soft Ring

Let R be a ring and (F,A) be a fuzzy soft set over *R*. Then (F,A) is said to be a fuzzy soft ring over R if and only if for each $a \in A$ and $x,y \in R$

(1) $F_a(x+y) \geq \min\{ F_a(x) , F_a(y)\}$

(2) $F_a(x^{-1}) \geq F_a(x)$

(3) $F_a(x.y) \geq \min\{ F_a(x) , F_a(y)\}$, where F_a is the fuzzy subset of R corresponding to the parameter $a \in A$.

3. EQUIVALENCE RELATION

Theorem 3.1 : Let $\mathscr{R} = \{ F_a : a \in A \}$. Consider the relation in $\mathscr{R}$ defined by $F_a \sim F_b$ if and only if $F_a(x) \geq F_a(y) \Leftrightarrow F_b(x) \geq F_b(y)$ for all $x,y \in R$. Then $\sim$ is an equivalence relation in $\mathscr{R}$

Definition3.2 : Level subset F_a^t of F_a

Let R be a ring and F_a be a fuzzy subset of R, then the level subset F_a^t is defined as

$$F_a^t = \{x \in R : F_a(x) \geq t \text{ where } t \in [0,1] \text{ and } t \leq F_a(0)\}$$

Theorem 3.3 : The level subset is a subring of R

Theorem3.4 : If $F_a , F_b \in [F_a]$,then there exist t_1 and $t_2 \in [0,1]$such that F_a^{t1} and F_b^{t2}

Examples 3.5 :

(1) $\mathscr{R}$ of $<Z_6,+_6,._6>$ has two equivalent classes ,$[F_a]$ and $[F_b]$, corresponding to the equivalence relation defined above.

$[F_a]$ consist of all fuzzy sets F_a on R such that 1,3 and 5 have same membership grade and 2 and 4 have membership grade greater than that of 1,3 and 5

$[F_b]$ consist of all fuzzy sets F_b on R such that 1,2,4 and 5 have same membership grade and 3 has membership grade greater than that of 1,2,4 and 5.

Different Level set of F_a are $\{0\}$, $\{0,2,4\}$ and Z_6

Different Level set of F_b are {0} , {0,3} and Z_6

(2) $\mathscr{R}$ of <Z_5,$+_5$,$._5$> has only one equivalent class corresponding to the equivalence relation defined above. Since <Z_5,$+_5$,$._5$> has no proper nontrivial subrings the only level sets are {0} and Z_5.

It is clear that $\mathscr{R}$ of <Z_p,$+_p$,$._p$> where p is a prime has only one equivalence class.

Definition 3.6 : Let R be a ring . (F,A) be a fuzzy soft ring defined over R. For each x ∈ R,

define $F_{\wedge a}(\mathrm{x}) = \min\{F_a(\mathrm{x}), F_b(\mathrm{x}), F_c(\mathrm{x}) \ldots\}$

and $F_{\vee a}(\mathrm{x}) = \max\{F_a(\mathrm{x}), F_b(\mathrm{x}), F_c(\mathrm{x}) \ldots\}$ where a,b,c.. . ∈ A.