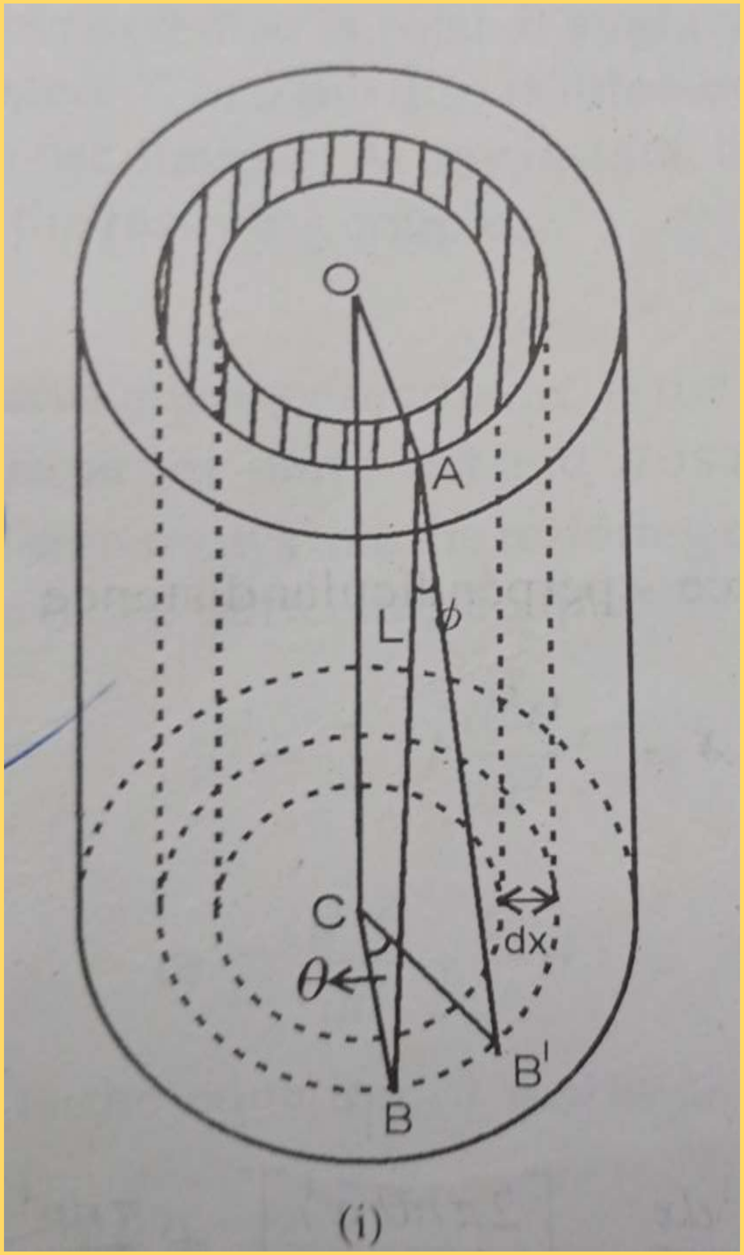
TWISTING COUPLE ON A CYLINDER

LALY A.S.
ASST. PROFESSOR
DEPT. OF PHYSICS
LITTLE FLOWER COLLEGE,
GURUVAYUR

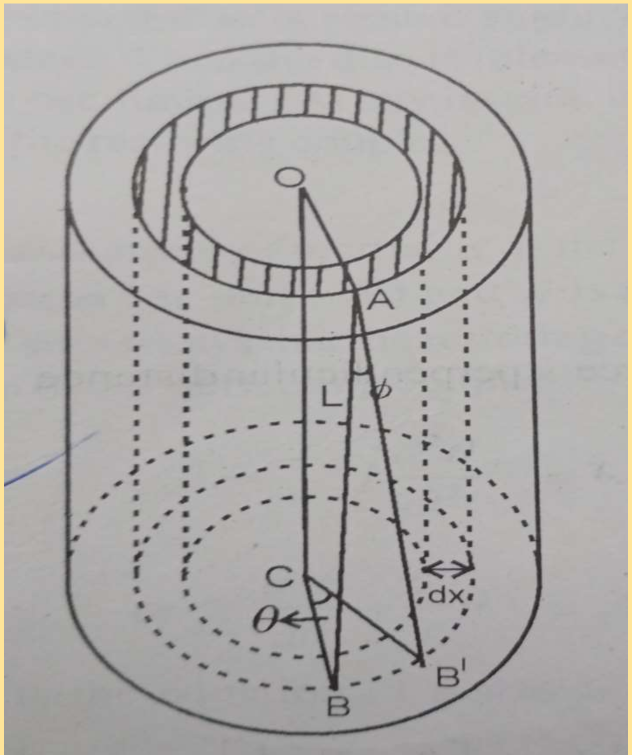


Consider a cylindrical rod of length ‘L’ and Radius ‘r’.

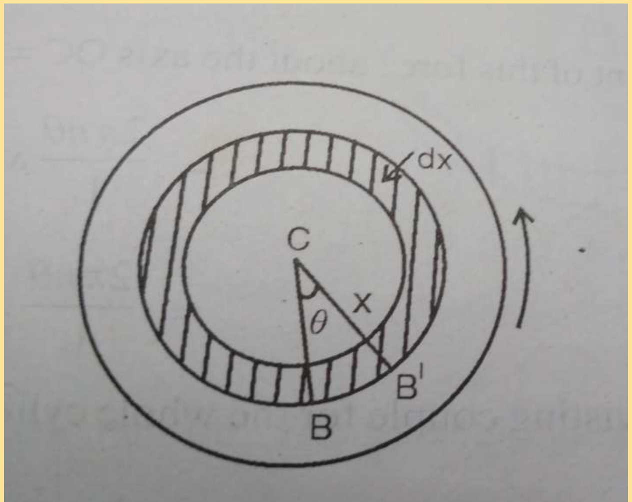
Its upper end is clamped and the rod is twisted by applying a couple to its lower end .

Due to the elasticity of the material ,a restoring couple is set up to oppose the twisting couple.

At equilibrium the two couples balance each other.



(i)



(ii)

Let us imagine the cylinder consists of large no. of thin coaxial cylindrical shells.

Consider one such shell of radius x and thickness dx and length L.

As the rod is twisted ,a line AB on the surface of the cylindrical shell takes up the position AB' through an angle BCB'=θ

The angle 'φ' is the angle of shear.

$BB' = x\theta = L\varphi$ or $\varphi = x\theta/L$

φ is maximum at the rim and zero at the axis

We have Rigidity modulus,

$$n = \frac{shearing\ stress}{shearing\ strain} = \frac{T}{\phi}$$

$$T = n\phi = \frac{nx\theta}{L}$$

$$Area\ of\ cross\ section\ of\ the\ shell = 2\pi x\ dx$$

$$Shearing\ force\ of\ the\ shell = T \times area\ of\ cross\ section$$

$$F = \frac{nx\theta}{L} 2\pi x\ dx = \frac{2\pi n\theta}{L} x^2\ dx$$

Moment of this force about the axis OC= Force×Perpendicular distance

$$= \frac{2\pi n\theta}{L} x^2 \, dx \, x \quad = \frac{2\pi n\theta}{L} x^3 \, dx$$

Total twisting couple for the whole cylinder,τ

$$\tau = \int_0^r \frac{2\pi n\theta \, x^3 \, dx}{L} = \left[\frac{2\pi n\theta}{L} \frac{x^4}{4} \right]_0^r = \frac{\pi n r^4}{2L}\theta$$

Let couple per unit twist be C and θ be the angle of twist,

The twisting couple $\tau = C\theta$

So,

$$C = \frac{\pi n r^4}{2L}$$

C is the couple per unit twist,n rigidity modulus of the material of cylinder,r radius of cylinder,L length of the cylinder

Workdone in twisting a wire through an angle θ

If “c” is the couple pe unit twist,then the couple required to twist the wire through an angle θ is cθ.

Then the work done in twisting further through a small angle dθ is

dw=cθ dθ

Hence the total work done in twisting a wire through an angle θ

$$w = \int_0^\theta c\theta \, d\theta = \frac{1}{2} c\theta^2$$

w=½ $c\theta^2$

.Calculate the work done in twisting a steel wire of radius 10^{-3}m and length 0.25m through an angle of 45^0.Given the rigidity modulus of the wire is 8×10^{10}N/m^2.

$$w = \frac{1}{2} c\theta^2, \quad C = \frac{\pi n r^4}{2L}$$

$r = 10^{-3}$m, $L = 0.25$m, $\theta = 45^0 = \pi/4$ radian $n = 8 \times 10^{10}$N/m^2

w=.1547J

THANK YOU....