Frames of reference

Prepared by
Ms. Anne Jose M
Department of Physics
Little Flower college, Guruvayoor

Position

- To locate the position of a body we need some arbitrary reference point.

  For convenience we take origin as the arbitrary reference point.

- The position of a particle at P from the reference point is represented by $\vec{r} = \overrightarrow{op}$
- In three-dimensional space the position of a particle is represented by the three space coordinates x, y and z.

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$

Where $\hat{\imath}, \hat{\jmath}, \hat{k}$ are unit vectors along the respective axes.

Rest And Motion

The state of rest or motion of a body is described relative to another body.

A body is said to be at rest if it does not change its position with time and if it changes its position continuously it is said to be in motion.

Since all the bodies in the universe are in motion, we cannot find any object which is absolutely at rest. But to find out whether a body is at rest or not, we compare it with a body which is apparently at rest.

The state of rest and motion are relative.

For example, a person sitting in a moving bus is at rest with respect to his co-passengers, but is in motion relative to another bodies or people on the road.

Frames of reference

It is that part of the world which we used to measure the motion of bodies.

When we are standing on a ground the ground is our frame of reference. The motion of other bodies are described with respect to that ground.

On the other hand if we are in a bus, bus is our frame of reference and everything will be related with that frame.

In order to describe a motion of a particle, a reference point (origin) and a reference direction or co-ordinate system is necessary. These two conditions are applied when we introduce the concept of frame of reference.

The system of coordinates used to describe the position of a particle in space is called a frame of reference. The simplest frame of reference is the cartesian coordinate system.

To describe an event like collision of bodies or bomb explosion in addition to the three space coordinates, time co-ordinate is also required. Frames of reference with space coordinates x,y,z and time co-ordinate t used to represent an event is called a space time frame of reference in cartesian co-ordinate system.

Basic concepts of Newton's Law of Motion

- The first law of motion of the Law of Inertia tell that a body which has no force acting on it will continue its state of rest or of uniform motion.
- The second law gives the concept of force F=ma and a force acting on a body produces acceleration.

Different types of frames of reference

Inertial frames of reference

Frames of reference in which the law of inertia holds true are called inertial frames of reference.

In an inertial frame, a body continues in its states of rest or of uniform motion along a straight line till external forces are applied.

It is a non accelerating frame of reference. That is either at rest or moving with constant velocity and newton's laws are valid .

Example: A system of fixed stars has been used as an inertial frame of reference, since they are at very large distance and can be thought of being free from the influence of other bodies.

A frame of reference on Earth is not inertial since it is rotating about its axis and revolving around the sun. These rotations will produce centripetal acceleration.

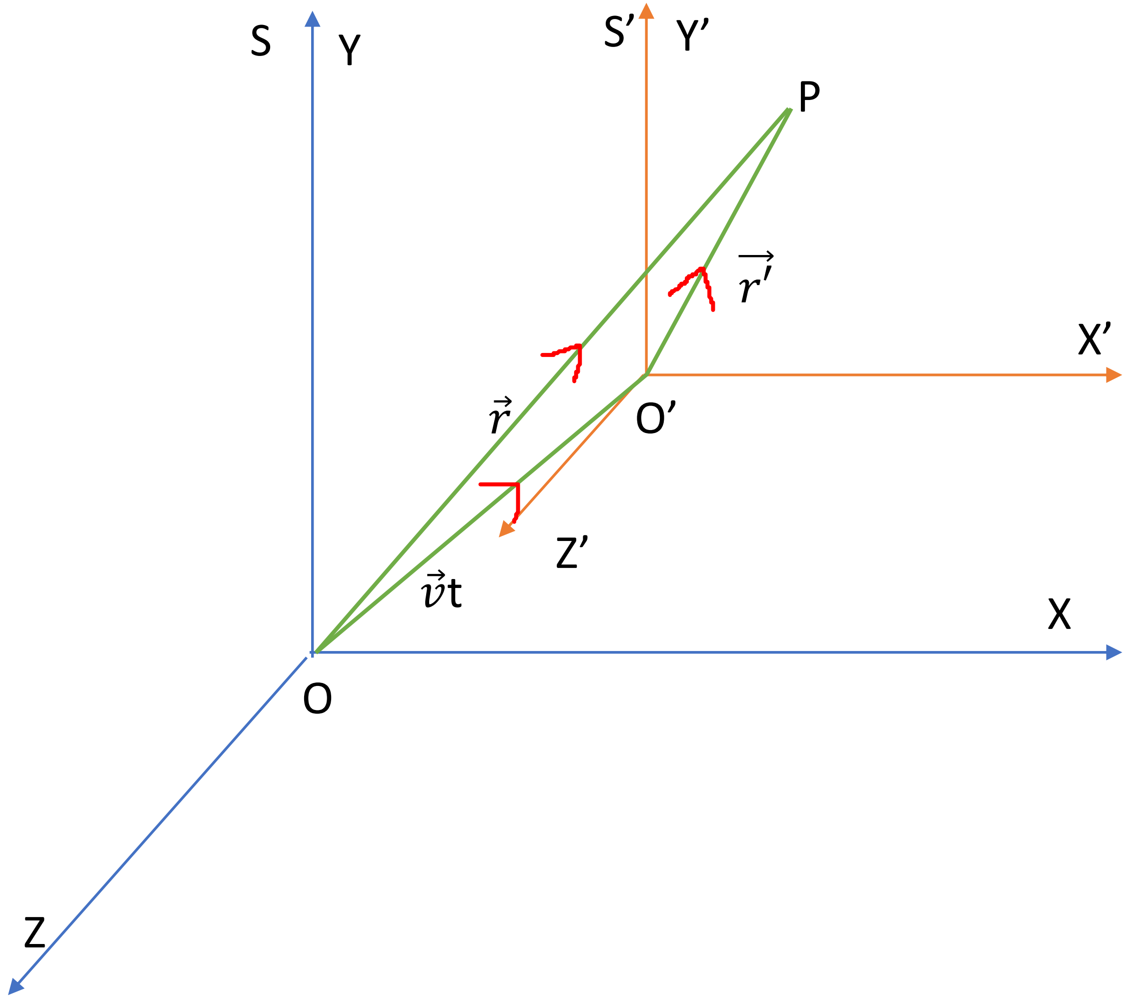
But often the co-ordinates system attached to Earth is considered to be an approximate inertial frame of reference. Scientist have given several explanations for this assumption.

1. Due to the slowness of rotation of the earth relative to fixed stars.
2. The studies of motion on earth frame are not much affected by the rotation and revolution of Earth.

All frames of reference which are moving with a constant velocity relative to an inertial frame of reference are also inertial.

Proof:

Consider two inertial frames of reference S and S'. Let S' be moving with a constant velocity 'v' with respect to S. Initially at time t=0, let their origins O and O' coincide. After a time 't', the position vector of a particle will be '**r**' in the frame S and **r'** in the frame S'.



In the figure, $\overrightarrow{OP} = \vec{r}$ and $\overrightarrow{O'P} = \overrightarrow{r'}$.

And $\overrightarrow{OP} = \overrightarrow{OO'} + \overrightarrow{O'P}$

ie, $\vec{r} = \vec{v}t + \overrightarrow{r'}$

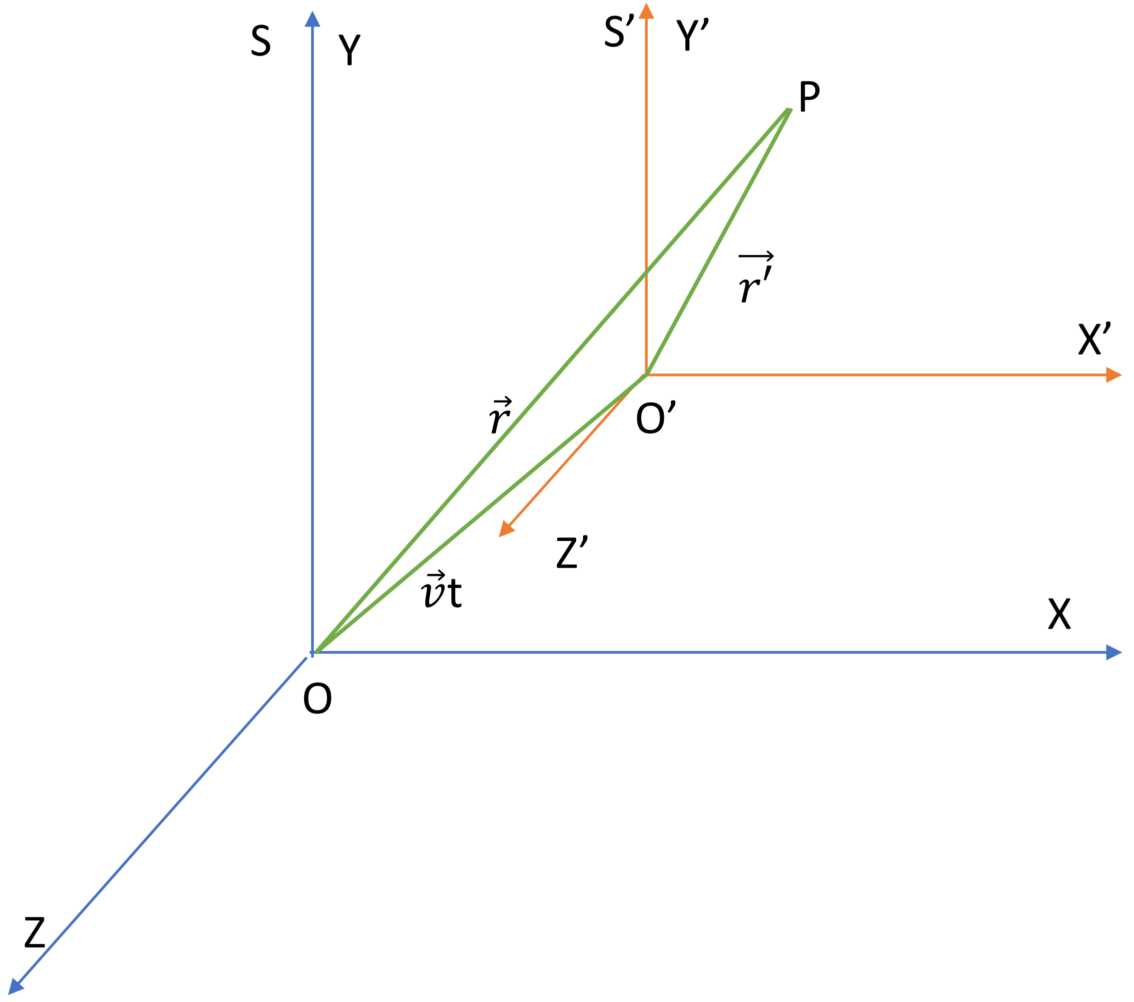
ie, $\overrightarrow{r'} = \vec{r} - \vec{v}t$

Differentiating w.r.t. t ,

$$\frac{d\overrightarrow{r'}}{dt} = \frac{d\vec{r}}{dt} - \vec{v}$$

Differentiating again w.r.t. t ,

$$\frac{d^2\overrightarrow{r'}}{dt^2} = \frac{d^2\vec{r}}{dt^2}$$ since v is constant.



$$\overrightarrow{a'} = \vec{a}$$

Thus, the acceleration of the particle in the two frames are equal.

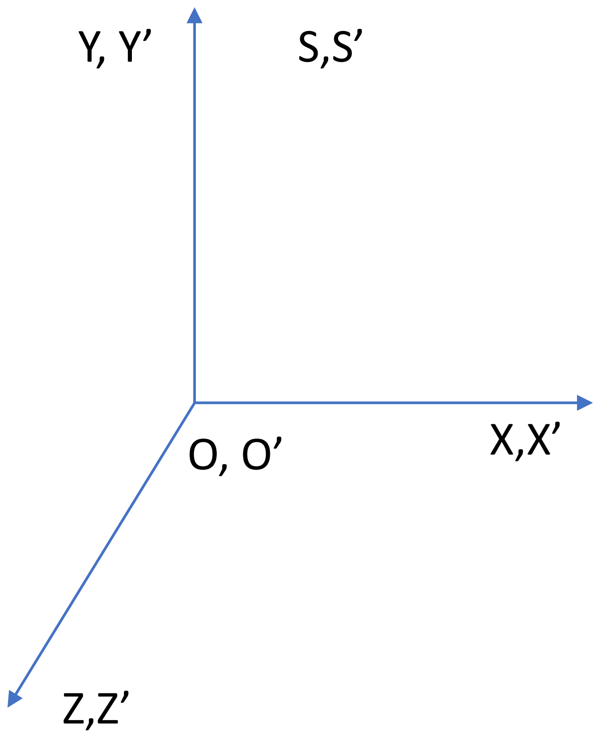
When the acceleration of a particle in the frame S is zero, the acceleration of the particle in a frame S’ moving with constant velocity is also zero. That is, if S is an inertial frame, S’ is also inertial.

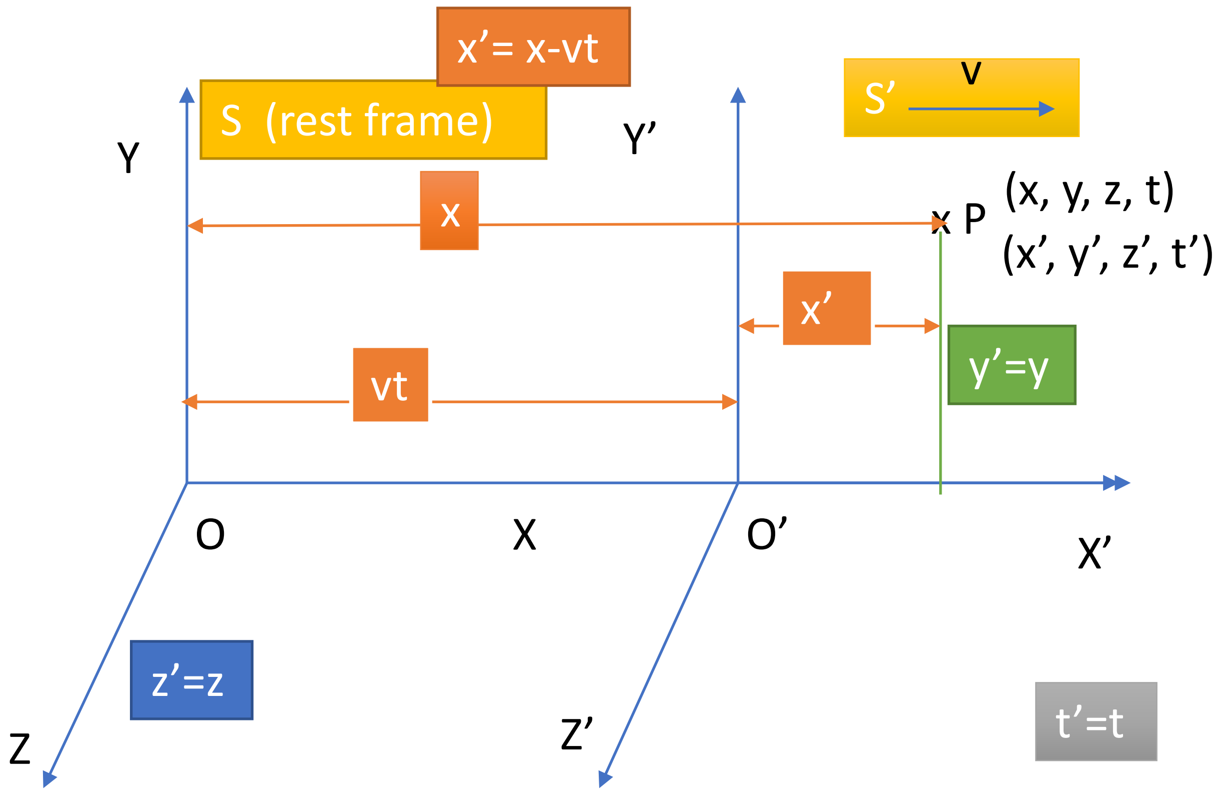
Transformation equations

A set of equations connecting space coordinates and time in two different frames of reference are called transformation equations.

Galilean Transformations

t=t'=0

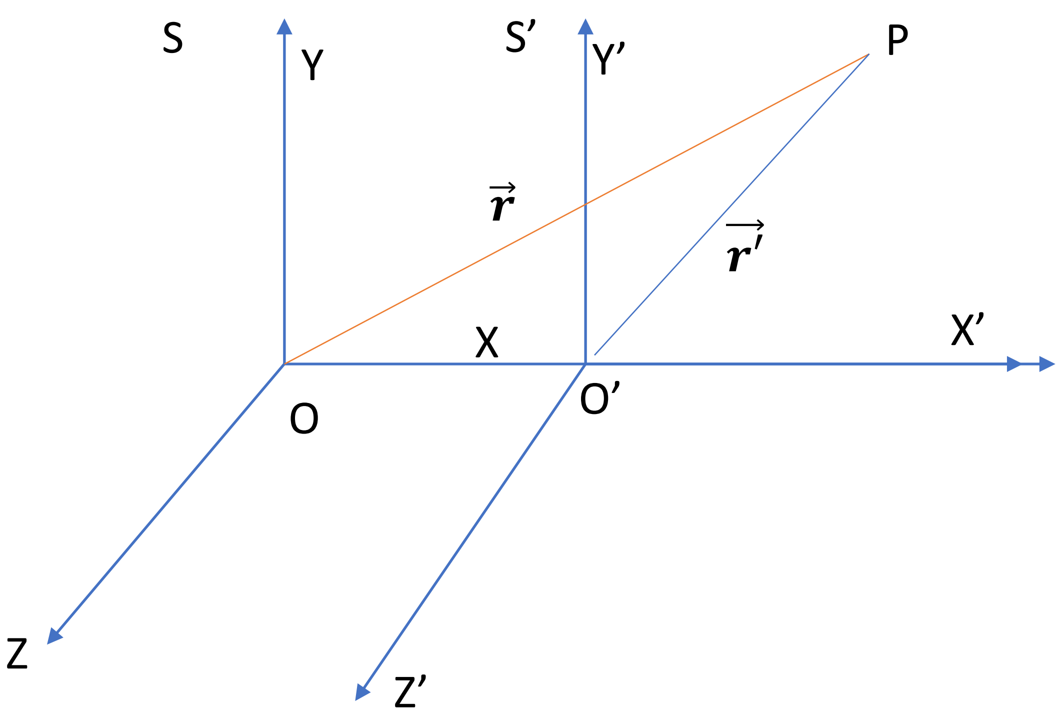




Galilean transformations

The set of equations relating coordinates of an event in two inertial frames of reference are called Galilean transformation equations.

Consider two inertial frames S and S' whose origins O and O' coincide at t=0. S'is moving with uniform velocity $\vec{v}$ relative to S along OX. The co-ordinate axes of S' are parallel to that of S. If two observers make observations of the same event in the two frames S and S', then there must be a relation between their measurement. Let an event P is represented by the co-ordinates (x',y',z',t') in S' and (x,y,z,t) in S. $\vec{r}$and $\overrightarrow{r'}$are the position vectors of P in S and S'.



From the figure, $\overrightarrow{OP} = \overrightarrow{OO'} + \overrightarrow{O'P}$

ie, $\vec{r} = \vec{v}t + \overrightarrow{r'}$

ie, $\overrightarrow{r'} = \vec{r} - \vec{v}t$

Therefore the transformations of space co-ordinates are

x’=x-vt

y’=y

z’=z

Since there is no motion along the Y and Z directions, the space coordinates remains the same.

Let t & t’ be the time recorded by the observers in S and S’ of an event at P, then, t’=t since time is independent of any particular frame of reference.

The equations

$x' = x - vt$

$y' = y$

$z' = z$

$t' = t$

are called Galilean transformation equations.

Inverse Galilean transformation equations

If an inertial frame S is moving with a velocity ‘v’ along the negative direction of x-axis with respect to S’ frame, then the transformation equations become

$x = x' + vt$

$y = y'$

$z = z'$

$t = t'$

These are known as inverse Galilean transformation equations.

Similar to co-ordinate transformation equations between two inertial frames, we can have transformation equations between physical quantities such as length, velocity, acceleration etc.

Transformation of length

Consider a rod has length L in the inertial frame S with end coordinates $x_1 and x_2$.

Then L= x_2- x_1 in One dimension.

If the end coordinates of the same rod in S' moving with a uniform velocity V are x_1' $and x_2'$,

then L'= x_2'- x_1'= $x_2 - vt$- (x_1-vt)= x_2- x_1

Hence L=L'

That is the length of the distance between two points is invariant under Galilean transformation

Transformation of velocity

Let v and v' be the velocity of a particle in the frame S and S' respectively.

Then $\vec{v} = \frac{\overrightarrow{dr}}{dt}$ and $\overrightarrow{v'} = \frac{\overrightarrow{dr'}}{dt}$

Taking the Galilean transformation equation $\overrightarrow{r'} = \vec{r} - \vec{V}t$,

And differentiating

$\frac{d\overrightarrow{r'}}{dt} = \frac{d\vec{r}}{dt} - \vec{V}$ ie, $\overrightarrow{v'} = \vec{v} - \vec{V}$

This equation shows that the velocity is not invariant under Galilean transformation. If the velocity of a body in a frame S is given, the velocity of the body in an inertial frame S' moving with a uniform velocity V is

$\overrightarrow{v'} = \vec{v} - \vec{V}$

Transformation of acceleration

Let $\vec{a}$ $and\overrightarrow{a'}$ be the accelerations of a particle in frame $\vec{S}$ and $\overrightarrow{S'}$ respectively.

We have $\overrightarrow{v'}$ = $\vec{v}$ - $\vec{V}$

Differentiating w.r.t. t, $\frac{d\overrightarrow{vi}}{dt} = \frac{d\vec{v}}{dt} - 0$

$\overrightarrow{a'}$= $\vec{a}$

Acceleration is invariant under Galilean transformation..

Problem 1: A train moving at 108km/hr passes a railway station at t=0. After 15 seconds, a man in the railway station see a bird flying along the track 1/2km away in the same direction of the train. What are the coordinates of the bird with respect to (a) man in the railway station (b) passenger in the train

Here, railway station is the S frame. Train is S frame. Position of bird at time 15s is taken as the event P. So we have to find out the coordinates of P in S and S'. ie (x,y,z,t) and (x',y',z',t').

Let the coordinates of the bird with respect to the man in the railway station is (x,y,z,t)=(500,0,0,15s)

To find the position coordinates of the bird with respect to a passenger in the train, we can use Galilean transformation.

x'=x-vt=500-(30x15)m=50m.

(Here, velocity V= 108km/hr is converted to m/s as V=$\frac{108\times1000}{60\times60}$=30m/s.)

ie, the coordinates of the bird with respect to a passenger in the train =(x',y',z',t')=(50,0,0,15)

A ship is moving at a speed 20 m/s relative to earth. Let a toy car be moving at a speed of 3m/s relative to ship in the direction of motion of the ship. Find the speed of the toy car with respect to earth.

Earth- S frame   ship-S' frame

Velocity of S' frame=velocity of ship V= 20m/s.

Velocity of particle in S frame= Velocity of toy car in Earth= $\vec{v}$ =?

Velocity of particle in S' frame= Velocity of toy car in ship= $\overrightarrow{v'}$ =3m/s

According to galilean transformation of velocity

$\overrightarrow{v'} = \vec{v} - \vec{V}$

$\vec{v} = \overrightarrow{v'} + \vec{V} = 3+20=23$m/s

From a sample of radioactive material which is at rest in the laboratory, two electrons are emitted in opposite direction. One of the electrons has a speed of 0.5c and the other has a speed of 0.6c with respect to an observer in the laboratory. What is the speed of one electron with respect to the other under Galilean transformation?

Let a frame S is attached with the laboratory which is at rest and another frame S' is attached with any one of the electrons. Here, with the electron of speed 0.5c and it is moving in the positive direction of x-axis.

ie, Velocity of S' wrt S is $\vec{V}$=0.5c

The speed of the other electron with respect to rest frame S $\vec{v}$ =-0.6c. Negative sign indicates that the observed electron is moving in the opposite direction of frame S'.

Velocity of the observed electron wrt the other electron= velocity of observed electron in frame S' = $\overrightarrow{v'}$= $\vec{v} - \vec{V}$ =-0.6c - 0.5c=-1.1c.

Thus we can see that velocities greater than speed of light are possible in Galilean transformation which is against special theory of relativity.

Invariance of an equation

By invariance of an equation, it is meant that the equation will have the same form when determined by two observers in the two frames. By applying Galilean transformation to an equation in one inertial frame, its new form in another inertial frame of reference will be obtained. If the two forms are the same then the equations are invariant under the Galilean transformation.

Thus we have seen that velocity is variant while length and acceleration are invariant under galilean transformation.

According to the galilean invariance hypothesis the basic laws of physics are invariant in all inertial frames of reference.

Non inertial frame of reference

A person sitting in a uniformly moving car does not feel anything unusual. But when the car is accelerated hard he feels ‘pushed back into the seat’. That is, in an accelerated frame of reference, the person experiences a backward force. At the same time, no force is experienced by him in an inertial frame of reference.

Thus we often find ourselves in situations in which bodies appears to be accelerating under the influence of some force even though no such force is actually acting on them violating Newton’s second law. This force is called as fictitious force. A fictitious Force is an Apparent force that acts on bodies in a non inertial frame of reference. This force does not arise from any physical interaction but from the acceleration of the frame result.

A frame of reference in which Newton’s Law of Inertia does not hold good is called a non inertial frame of reference. In this frame, bodies experience fictitious force.

Examples:

- A frame of reference attached on a freely falling elevator
- Frames of reference following curved paths

Fictitious force

Consider two frames S and S'.

S is an inertial frame and S' is moving with an acceleration $\overrightarrow{a_0}$ relative to S.

Consider a particle at rest in frame S. This particle will have an acceleration - $\overrightarrow{a_0}$ with respect to S'.

If the particle in frame S is having an acceleration $\vec{a}$, then the acceleration of the particle with respect to frame S' is

$\overrightarrow{a'}$= $\vec{a}$- $\overrightarrow{a_0}$

If m is the mass of particle, which is a constant in the two frames of reference, then

$\overrightarrow{ma'}$= m$\vec{a}$- m$\overrightarrow{a_0}$

$\overrightarrow{F'}$= $\vec{F}$- m$\overrightarrow{a_0}$

F'= $\overrightarrow{ma'}$ is the force experienced by a particle in the non-inertial frame

F= m$\vec{a}$ is the actual force on the particle in the frame S.

When $\vec{F}$=0, $\overrightarrow{F'}$= - m$\overrightarrow{a_0}$.

That is, Even if the force acting on the particle in the frame S is zero, a force appears to act on the particle with respect to S'. This force is called Fictitious force or Pseudo force.

Thus fictitious force or pseudo force is an apparent force that acts on a particle due to the acceleration of the frame in which it is placed.

Example: Motion in a lift.

Consider a man having mass m uses a lift to go to the top of a building.

Let the lift is moving with an acceleration a_0 upwards. Then it is a non-inertial frame.

Hence he will experience a fictitious force-m a_0.

The total force acting on the man in the lift is =-mg- m a_0 =-m(g+ a_0)

(mg is the weight of the man. Here upward direction is taken as positive)

The total force is acting downwards. Thus his weight appears to be increased.

Suppose he returns to the ground using the same lift and a_0 is the acceleration of the frame.

Then the total force experienced by him= -mg-[m(- a_0)]

=-mg+m a_0=-m(g- a_0)

In this frame, his weight appears to have decreased.

If the lift is moving with constant velocity, then the acceleration of the lift is zero. Hence no fictitious force acts. So the force on the man =-mg.

Now the lift is falling freely under gravity. Then a_0=g.

The total force acting on the man = -mg+mg =0.

That is he feels weightless.

Fictitious Centrifugal force

Consider a uniformly rotating frame of reference. Let ω be its angular velocity about a fixed axis at the origin of the coordinate system. In this frame, let a particle of mass m be at rest and the observed acceleration of the particle within this frame is zero. The particle will experience a force called centripetal force towards the axis of rotation and is given by $-m\omega^2 r$, where $\vec{r}$ is the radial distance directed outward from the centre.

Now $\left|\overrightarrow{F'}\right| = \left|\vec{F}\right| + \left|\overrightarrow{F_0}\right|$

Since the observed acceleration of the particle in the non-inertial frame is zero, $\overrightarrow{F'}=0$

$\therefore 0 = -m\omega^2 r + \left|\overrightarrow{F_0}\right|$

$\therefore \quad \left|\overrightarrow{F_0}\right| = m\omega^2 r$

Hence the fictitious force acting on a particle at rest in a rotating frame of reference= $m\omega^2 r$

This fictitious force is called the centrifugal force, which is directed away from the centre. It is acting only in rotating frames.

Reactive Centrifugal Force

Reactive centrifugal force is the reaction force to a centripetal force

Let a stone of mass m is attached at one end of a string and by holding the other end of string it is whirled around in horizontal plane. The necessary centripetal force on the stone is given by the Tension T of the string. The stone exerts equal and opposite force on the string. Thus the reactive force is exerted by the stone. This is called reactive centrifugal force. Thus centripetal force is acting on a particle as it revolves around. The reactive centrifugal force is the force exerted by the particle on some other object when it is in circular motion.

$$\vec{F} = \vec{T} = m\omega^2 r$$

This means that in the non- inertial frame, the centrifugal force is balanced by the inward tension in the string.

In a rotating frame, the force experienced by the particle=centripetal force + centrifugal force = 0

A rocket is moving upwards with an acceleration 2g. Compute the effective weight of a man standing in it, when his actual mass is 70kg

$A = 2g$

$m = 70kg$

Effective weight $= -mg - mA = -70g - 70 \times 2g = -70g - 140g = -210g$

Find the fictitious force and total force acting on a body of mass 3 kg relative to a frame moving vertically upwards from earth with an acceleration of 10 m/sec^2.(g= 9.8m /sec^2)

The fictitious force= -mA = -3kg x 10 = -30N, acts down wards

The total force= -mg –ma =   -3x9.8 -3x10 = -59.4 N**, acts down wards**

An apple of mass 250 gm falls freely under gravity. What is the fictitious force and total force on it, with reference to a frame moving with an acceleration of 3 m/sec^2 in the downwards direction.

Fictitious force=-mA= -0.250 x -3 = 0.75 N upwards

Total force= -mg-m(-A)=-mg + mA= - 0.250x9.8 + 0.250x 3 = -1.7 N acts downwards.

A rocket is moving upwards with a velocity 1500m/s w.r.t. ground. The burnt gases are ejected in the opposite direction with a velocity of 500m/s w.r.t. the ground. Determine the velocity of gases w.r.t. rocket

Ground – S frame

Rocket- S' frame

Burnt gases- P

Velocity of rocket V=1500m/s

Velocity of gas wrt ground v=-500m/s

Velocity of gas w.r.t.rocket, v'= v-V=-500-1500= -2000m/s

A train is moving with a velocity 90km/hr passes a railway station. Twenty seconds later an explosion takes place on the track 1km away from the station in the same direction of the train. Find the position coordinate of the explosion as measured by an observer at the station and by the driver of the train.

- Railway station – S Train – S' Explosion – P

Velocity of train wrt railway station = velocity of S' wrt S= V= 90km/hr=90x1000/3600 m/s=25m/s.

t=20sec

x=1km = 1000m

Postion coordinate of explosion wrt an observer at the station=(x,y,z,t)=(1000, 0, 0, 20)

Postion coordinate of explosion wrt an observer at train = (x',y',z',t') =(x-vt, 0, 0, 20)= (1000-25x20, 0,0, 20) = (500, 0,0, 20)

A rocket is moving downwards with an acceleration 2g. Compute the effective weight of a man standing in it, if his actual weight is 80kg.

A=2g downwards

M=80kg

Effective weight= -mg+mA = -80g+80x2g=-80g+160g=80g =80kgwt

An astronaut weighs 350kg in a moving rocket. If the rocket is going upwards with an acceleration 4g, find the weight of the astronaut in the laboratory

$F' = -mg - mA$

$m = ?$

$350g = -m(g + 4g) = -m \times 5g$

$m = -350g/5g = -70$

$mg = 70g$

Coriolis force

If a body is moving with a linear velocity $\vec{v}$ in a non-inertial frame rotating with angular velocity $\vec{\omega}$, then

Force acting on the particle in rotating frame is

F'=F-2m($\vec{\omega} \times \vec{v}$) $- m\vec{\omega} \times (\vec{\omega} \times \vec{r})$

-2m($\vec{\omega} \times \vec{v}$)-Coriolis force. It acts at right angle to the axis of rotation and to the velocity of the particle in the rotating frame.

$-m\vec{\omega} \times (\vec{\omega} \times \vec{r})$- centrifugal force directed away from centre.

$\overrightarrow{F_0}$=-2m($\vec{\omega} \times \vec{v}$) $- m\vec{\omega} \times (\vec{\omega} \times \vec{r})$

Coriolis force and centrifugal forces are fictitious forces acting on a particle in non-inertial frame.

Coriolis force acts on a particle in motion whereas centrifugal force acts on a particle at rest.

Coriolis force is a fictitious force which acts on a particle only if it is in motion with respect to the rotating frame.

Earth rotates from west to east about its own axis. So it is a rotating non-inertial frame. If a body on the surface of earth is in motion w.r.t. earth, then Coriolis force acting on the body is $-2m(\vec{\omega} \times \vec{v})$ where v is the velocity of body w.r.t. earth.

Coriolis force causes a moving particle in the northern hemisphere to deflect towards the right of its path . In southern hemisphere, the deflection is towards the left of the path.

The freely falling bodies deviate towards east direction in either of the hemisphere of earth.

A mass of 0.1 kg is moving with linear velocity of 2.5 m/sec normal to the axis *rotation in a rotating frame of reference. The mass is kept at a distance of 0.2 m from the axis of rotation. Determine the coriolis force and centrifugal force experienced by the mass.*

coriolis force =-2m$(\vec{\omega} \times \vec{v})$=-2m ωv since sin 90=1

ω=v/r=2.5/0.2=12.5rad/s

coriolis force =-2m$(\vec{\omega} \times \vec{v})$=-2m ωv=-6.25N

centrifugal force= $m\omega^2 r$=3.125N

A stone of mass 1kg tied at the end of a 1m long string makes 5 revolutions per second. Calculate the tension in the string.

Tension in the string=centripetal force = $m\omega^2 r$

m=1kg

$\omega = 2\pi N = 2 \times \pi \times 5 = 10\pi$

r=1m

Tension, T = $1 \times (10\pi)^2 \times 1 = 100\pi^2 N$

Find the fictitious acceleration of the sun relative to a frame rotating with earth. Distance between the earth and sun=1.5$\times 10^{11} m$.

$$F' = F - 2m(\omega \times v) - m(\omega \times \omega \times r)$$

Fictitious force=$-2m(\omega \times v) - m(\omega \times \omega \times r)$

Fictitious acceleration=$-2(\omega \times v) - (\omega \times \omega \times r)$

Since $\quad v = -(\omega \times r)$,

Fictitious acceleration= $2(\omega \times \omega \times r) - (\omega \times \omega \times r)$ =$(\omega \times \omega \times r)$

= $-\omega^2 r$ = -$\left(\frac{2\pi}{T}\right)^2 r$ = -$\left(\frac{2\pi}{24\times60\times60}\right)^2 \times 1.5 \times 10^{11}$ =-7.8 $\times 10^2 m/s^2$

A massless string pulls a mass of 60kg upwards against gravity. The string would break if subjected to a tension greater than 780N. Find the maximum acceleration with which mass can be moved upwards.

M=60kg

Maximum tension, T= 780N

T=mg+mA = m(g+A)

g+A=T/m=780/60=13

A=13-g=13-9.8=3.2 m/ s^2